WORKED SOLUTIONS

VE2.4: VECTORS: INTERSECTING PLANES

Question

Find (a) the angle between and (b) parametric equations for the line of intersection of the two planes

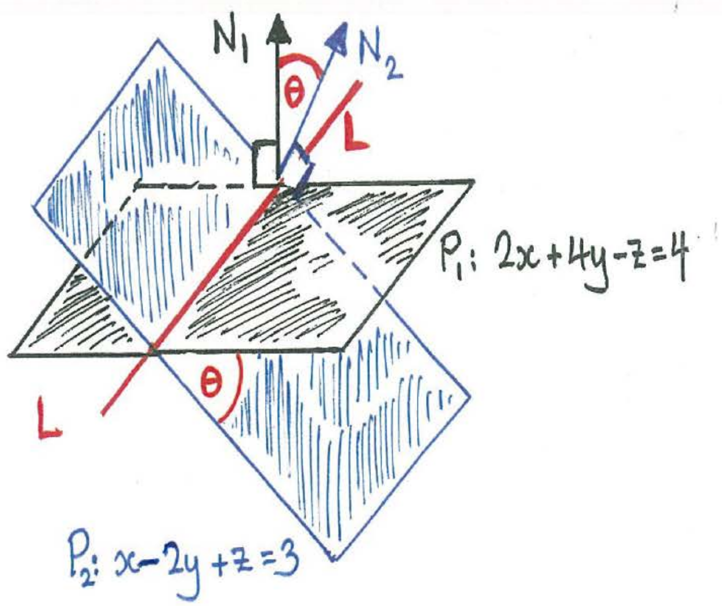
$P_1 : 2x + 4y - z = 4$ and $P_2 : x - 2y + z = 3$

Worked Solution

(a) The angle θ between planes P_1 and P_2 is the same as the angle between their normals N_1 and N_2 since each normal is perpendicular to its plane. See diagram below

$P_1 : 2x + 4y - z = 4$
$N_1 = 2i + 4j - k$

$P_2 : x - 2y + z = 3$
$N_1 = i - 2j + k$



Using scalar (dot) product $N_1 \cdot N_2 = |N_1||N_2| \cos\theta$

$$\Rightarrow \cos\theta = \frac{N_1 \cdot N_2}{|N_1||N_2|}$$

where $|N_1| = \sqrt{(2)^2+(4)^2+(-1)^2} = \sqrt{21}$ and $|N_2| = \sqrt{(1)^2+(-2)^2+(1)^2} = \sqrt{6}$

Hence $\cos\theta = \dfrac{(2,4,-1)\cdot(1,-2,1)}{\sqrt{21} \times \sqrt{6}} \Rightarrow \theta = \cos^{-1}\dfrac{-7}{\sqrt{126}} = 128°$ //

NOTE: 2 answers are possible: 128° and 52° (180−128).

(b) Planes P_1 and P_2 intersect along the line **L**. See diagram
Substituting $z=t$ into P_1 and P_2, where t is a real number.

$$\begin{array}{lcr} 2x+4y-t=4 & \rightarrow & 2x+4y-t=4 \quad + \\ x-2y+t=3 & \xrightarrow{\times(-2)} & \underline{-2x+4y-2t=-6} \\ & & 8y-3t=-2 \\ & & \Rightarrow y=-\frac{1}{4}+\frac{3}{8}t \end{array}$$

Substituting for y into $x-2y+t=3$
$x-2\left(-\frac{1}{4}+\frac{3}{8}t\right)+t=3 \Rightarrow x=\frac{5}{2}-\frac{1}{4}t$

Hence, parametric equations are:
$x=\frac{5}{2}-\frac{1}{4}t, \quad y=-\frac{1}{4}+\frac{3}{8}t, \quad z=t$ //