RMIT UNIVERSITY

WORKED SOLUTIONS

VE2.2: VECTORS: INTERSECTING LINES

Question

Find the point of intersection of the two lines

$$L_1 : \frac{x-10}{-2} = \frac{y+2}{2} = \frac{z-5}{1} \quad \text{and} \quad L_2 : \begin{array}{l} x = -2-2t \\ y = -6-4t \\ z = 6+3t \end{array}$$

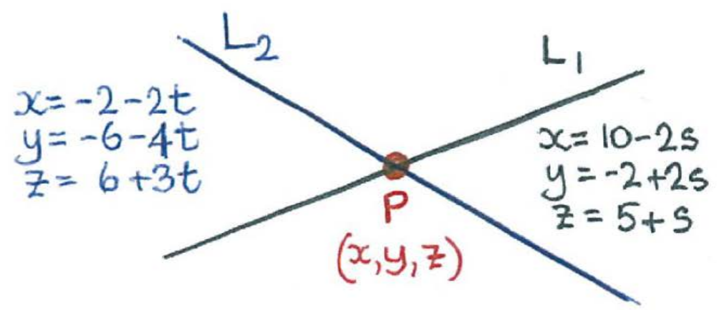
Worked Solution

Write L_1 as parametric equations

$$\begin{array}{lll} x-10 = -2s & \Rightarrow & x = 10-2s \quad \text{---}(1) \\ y+2 = 2s & \Rightarrow & y = -2+2s \quad \text{---}(2) \\ z-5 = s & \Rightarrow & z = 5+s \quad \text{---}(3) \end{array}$$

NOTE : Two different lines L_1 and L_2 must have two different parameters 's' and 't'.

To find the intersection of these 2 lines we find a point $P(x, y, z)$ that is common to these lines.



	L_1		L_2	
x :	$10 - 2s$	$=$	$-2 - 2t$	--- ④
y :	$-2 + 2s$	$=$	$-6 + 4t$	--- ⑤
z :	$5 + 5s$	$=$	$6 + 3t$	--- ⑥

Solving for 't' in ④ gives $t = -6 + s$

Solving for 't' in ⑤ gives $t = 1 + \frac{1}{2}s$

Hence : $-6 + s = 1 + \frac{1}{2}s \Rightarrow s = 14.$

Substituting $s = 14$ into

Equation ① : $x = 10 - 2(14) = -18$
Equation ② : $y = -2 + 2(14) = 26$
Equation ③ : $z = 5 + 14 = 19$

Hence $P(-18, 26, 19)$ is the point of intersection of the lines L_1 and L_2.

NOTE :

- Equations ④ and ⑤ were used to solve for 't', however equations ⑤ and ⑥ could have also been used
- Solving for 's' instead of 't' would also have resulted in the same answer.
- Check your answer by substituting $x = -18$, $y = 26$, $z = 19$ into L_1 and solving for 's'. The answer should be consistent, i.e. $s = 14$ for equations ①, ② and ③