RMIT UNIVERSITY

WORKED SOLUTIONS

VE2.3: VECTORS: EQUATION OF A PLANE

Question

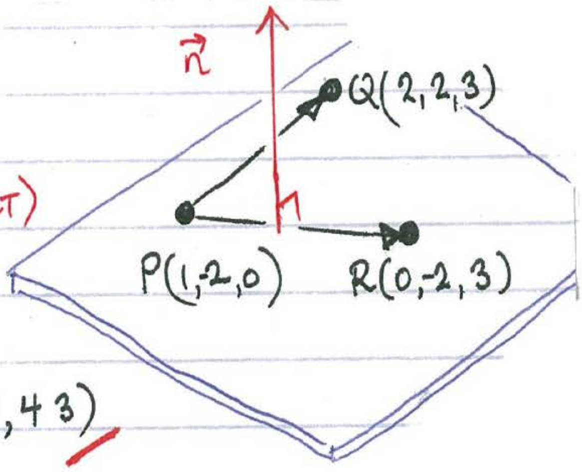
Find the equation of a plane that contains the points $P(1,-2,0)$, $Q(2,2,3)$ and $R(0,-2,3)$

Worked Solution

To find the equation of a plane we need

- a point P on the plane.
- a vector $\vec{n}$ normal to the plane.

To find the normal to the plane:



$\vec{n} = \vec{PQ} \times \vec{PR}$ (CROSS PRODUCT)

where $\vec{PQ} = \vec{OQ} - \vec{OP}$

$\vec{PQ} = (2,2,3) - (1,-2,0) = (1,4,3)$

and $\vec{PR} = \vec{OR} - \vec{OP}$

$\vec{PR} = (0,-2,3) - (1,-2,0) = (-1,0,3)$

Cross Product

$$\vec{n} = \vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 1 & 4 & 3 \\ -1 & 0 & 3 \end{vmatrix} = (12-0)i - (3+3)j + (0+4)k$$

$$= 12i - 6j + 4k = (12,-6,4)$$

The equation of a plane is given by

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

where $\vec{n} = (\vec{a}, \vec{b}, \vec{c}) = (12, -6, 4)$ normal to plane

and $P = (x_0, y_0, z_0) = (1, -2, 0)$ a point on the plane.

Substituting into this equation gives:

$$12(x-1) + -6(y-(-2)) + 4(z-0) = 0$$

$$\Rightarrow 12x - 6y + 4z = 24 \quad \Rightarrow \quad \underline{6x - 3y + 2z = 12}$$

NOTE: Revision of Cross Product

For vectors $\vec{a} = a_1 i + a_2 j + a_3 k$
and $\vec{b} = b_1 i + b_2 j + b_3 k$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$$

$$= (a_2b_3 - a_3b_2)i - (a_1b_3 - a_3b_1)j + (a_1b_2 - a_2b_1)k$$