RMIT UNIVERSITY

WORKED SOLUTIONS

VE2.1: VECTORS: EQUATION OF A LINE

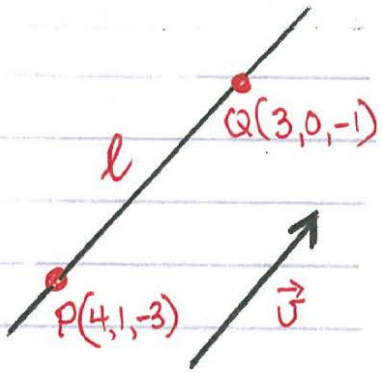
Question

Find (a) parametric and (b) symmetric equations of the line l through the points $P(4,1,-3)$ and $P(3,0,-1)$.

Worked Solution

To find the equation of a line we need:
- a point P on the line.
- a vector $\vec{v}$ parallel to the line.

Line l is parallel to vector $\vec{v}$ which is given by:



$\vec{v} = \vec{PQ} = \vec{OQ} - \vec{OP}$

$= (3,0,-1) - (4,1,-3)$

$\Rightarrow \vec{v} = (-1,-1,2)$

(a) The Parametric Equations of a line are

$x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$

where $\vec{v} = (a,b,c)$ and $P = (x_0, y_0, z_0)$ is a point on the line

$\vec{v} = (-1,-1,2)$ $P = (4,1,-3)$

(we could also have chosen Q)

Hence, $x = 4 + (-1)t \qquad y = 1 + (-1)t \qquad z = -3 + 2t$

$x = 4 - t \qquad y = 1 - t \qquad z = -3 + 2t$

(b) Symmetric Equations of a line

We may eliminate 't' from the parametric equations

$$x = x_0 + at \qquad y = y_0 + bt \qquad z = z_0 + ct$$

to obtain the following symmetric (or Cartesian) equations of the line:

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

Since $x = 4 - t$, $y = 1 - t$, $z = -3 + 2t$

and solving for 't', the symmetric equations of this line are

$$\frac{x - 4}{-1} = \frac{y - 1}{-1} = \frac{z + 3}{2}$$

NOTE: If one of the numbers a, b or c is zero, we use the remaining two equations to eliminate the parameter 't'

eg. if $a = 0$ from our question above then

$$x = 4, \quad \frac{y - 1}{-1} = \frac{z + 3}{2}$$

would be our symmetric equations of the line.