RMIT UNIVERSITY

WORKED SOLUTIONS

VE2.5: VECTORS: DISTANCE FROM A POINT TO A PLANE

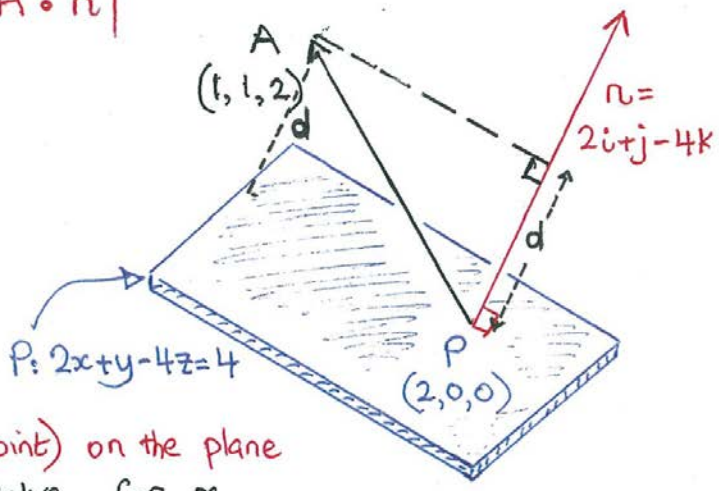
Question

Find the distance of the point A (1,1,2) to the plane $2x + y - 4z = 4$.

Worked Solution

$$d = |\vec{PA} \cdot \hat{n}|$$

d is length of vector projection of $\vec{PA}$ on to n
$\vec{PA}$ is the vector from point P on the plane to A
$\hat{n}$ is the unit normal to the plane.



Find a point P (any point) on the plane
Let $y = z = 0$ and solve for x
$2x + 0 - 4(0) = 4 \Rightarrow x = 2$ Hence $P = (2,0,0)$

Find $\vec{PA}$

$$\vec{PA} = \vec{OA} - \vec{OP} = (1,1,2) - (2,0,0) = -i + j + 2K = (-1,1,2)$$

Find the normal $\vec{n}$ to the plane

Since $2x + y - 4z = 4$, $n = 2i + j - 4K = (2,1,-4)$

Hence $\hat{n} = \frac{n}{|n|} = \frac{(2,1,-4)}{\sqrt{2^2+1^2+(-4)^2}} = \frac{1}{\sqrt{21}}(2,1,-4)$

Substitute for $\vec{PA}$ and $\hat{n}$ into $d = |\vec{PA} \cdot \hat{n}|$

$$d = \left|(-1,1,2) \cdot \frac{1}{\sqrt{21}}(2,1,-4)\right| = \frac{1}{\sqrt{21}}|-2+1-8| = \frac{1}{\sqrt{21}}|-9| = \frac{9}{\sqrt{21}} \times \frac{\sqrt{21}}{\sqrt{21}} = \frac{3}{7}\sqrt{21}$$