RMIT UNIVERSITY

VE1.3: VECTOR PRODUCT

The vector or cross product of two vectors **a** and **b** is the vector **a** × **b**, which is perpendicular to **both a and b** and is given by

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

The **magnitude** of **a** × **b** is given by $|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$ where θ is the angle between **a** and **b**.

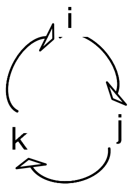
The **direction** of **a** × **b** is perpendicular to both **a** and **b** in the direction which your thumb would point if the fingers of your right are curled from **a** to **b**.

If **a** is parallel to **b** then **a** × **b** = 0. ($\sin 0^0 = 0$)

$$\mathbf{i}\times\mathbf{j} = \mathbf{k}, \quad \mathbf{j}\times\mathbf{k} = \mathbf{i}, \quad \mathbf{k}\times\mathbf{i} = \mathbf{j}$$
$$\mathbf{i}\times\mathbf{k} = -\mathbf{j} \quad \mathbf{k}\times\mathbf{j} = -\mathbf{i} \quad \mathbf{j}\times\mathbf{i} = -\mathbf{k}$$
$$\mathbf{i}\times\mathbf{i} = \mathbf{j}\times\mathbf{j} = \mathbf{k}\times\mathbf{k} = 0$$

The diagram at the right may be useful. The cross product is positive in the direction of the arrows, negative in the opposite direction.

For example, $\mathbf{k}\times\mathbf{i} = \mathbf{j}$ and $\mathbf{i}\times\mathbf{k} = -\mathbf{j}$



If **a** is perpendicular to **b** then $|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|$ ($\sin 90^0 = 1$)

The **order of multiplication is important.**

$\mathbf{a}\times\mathbf{b} \neq \mathbf{b}\times\mathbf{a}$ (the cross product is not commutative)
$\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$

Examples

1. Find a × b if a = 2i +3j + k and b = 5j +3k

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 1 \\ 0 & 5 & 3 \end{vmatrix} = (9-5)\mathrm{i} - (6-0)\mathrm{j} + (10-0)\mathrm{k}$$

$\mathbf{a}\times\mathbf{b}$ = 4i – 6j + 10k

2. Find **a** × **b** if **a** = (2,1,1) and **b** = (–2,4,0)

$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 1 \\ -2 & 4 & 0 \end{vmatrix} = (0-4)\mathbf{i} - (0+2)\mathbf{j} + (8+2)\mathbf{k}$$

$\mathbf{a}\times\mathbf{b}$ =– 4i – 2j + 10**k**

3. Find **a** × **b** if **a** = (2,1,1) and **b** = (8,4,4)

Because **a** = 4**b**, **a** is parallel to **b** therefore **a** × **b** = **0**

Exercise 1

Calculate the following.

1. j × k
2. i × 4i
3. (2i + 3j – k) × (3j + 2k)
4. 3j × 5i
5. (i – 3j + k) × (2i + j – k)

Exercise 2

Find a unit vector perpendicular to both (**i** – **k**) and (**i** + 3**j** – 2**k**).

Answers

Exercise 1

1. **i**
2. 0
3. 9i – 4j + 6k
4. – 15k
5. 2i + 3j + 7k

Exercise 2

$$\frac{(3,1,3)}{\sqrt{19}}$$