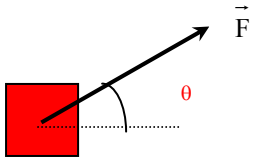
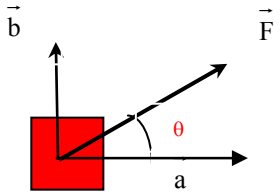
PV1.3: RESOLUTION OF VECTORS

Vector Components

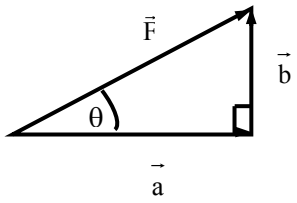
Force is a vector. As with other types of vectors (displacement, velocity), a force vector can be thought of as being the sum of several others. Imagining that a single force is made up of several forces can greatly simplify calculations, particularly using two imaginary ones at right angles to each other.

For instance, if you pull on an object with a force $\vec{F}$ (see diagram below left), the effect is the same as two other people pulling with forces $\vec{a}$ and $\vec{b}$ at right angles to each other (see diagram below right).





The vector triangle showing the force $\vec{F}$ and its component vectors $\vec{a}$ and $\vec{b}$ is shown below.



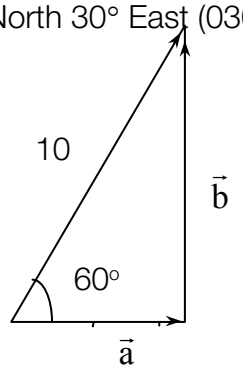
The force $\vec{F}$ can be imagined as having two parts, or **components,** $\vec{a}$ and $\vec{b}$ that are at right angles to each other. $\vec{a}$ is usually called the horizontal component and $\vec{b}$ the vertical component of $\vec{F}$. The components are often called **rectangular components** because they are at right angles to each other.

For the diagram above, it can be seen that

$$\sin\theta = \frac{b}{F} \quad \text{so} \quad b = F\sin\theta \qquad \text{and} \qquad \cos\theta = \frac{a}{F} \quad \text{so} \quad a = F\cos\theta$$

Example 1:

Calculate the rectangular components of a force of 10 N that acts in a direction of North 30° East (030°T)



The force and its rectangular components are shown. One component, $\vec{a}$, is acting to the east, the other, $\vec{b}$, to the north.

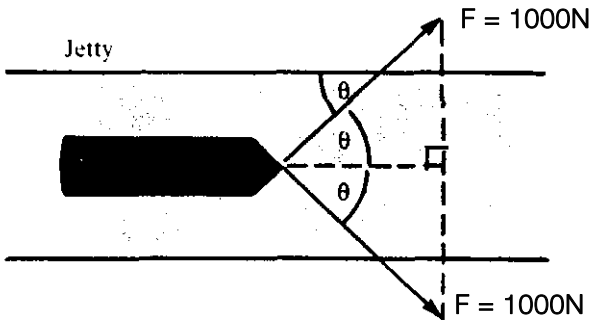
$$\vec{a} = \vec{F}\cos\theta = 10\cos 60° = 5\text{N}$$

$$\vec{b} = \vec{F}\sin\theta = 10\sin 60° = 8.7\text{N}$$

Thus, the 10N force can be resolved into two rectangular components: 5 N to the East and 8.7 N to the North.

Example 2:

A small boat is being towed at constant speed along a canal by two men walking along the banks on opposite sides pulling with equal forces of 1000N at equal angles on ropes attached to the boat. = 60º. Calculate the force each man pulls on the rope (a) parallel to the bank, and (b) at right angles to the bank.



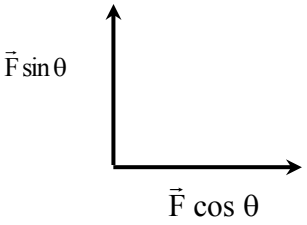
The 1000N force in each rope has two components:

(a) a component force **parallel** to the bank of

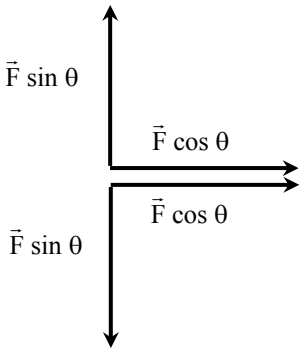
$\vec{a} = \vec{F}\cos\theta = 1000 \times \cos 60^\circ = 500\text{N}$

(b) a component force at **right angles** to the bank of

$\vec{b} = \vec{F}\sin\theta = 1000 \times \cos 30^\circ = 866\text{N}$



Note that the two components at right angles to the bank are equal in size, but are in opposite directions, so they cancel each other. So the boat will only move parallel to the banks.



The two components parallel to the banks are in the same direction so the force pulling the boat forwards is $2 \times \vec{F}\cos\theta = 2 \times 500 = 1000\text{N}$.

The balancing force causing the boat to move at constant speed is the friction or drag of the boat through the water acting in the opposite direction.

Exercise

1. What is the horizontal component of a force of 11N acting at 60º to the horizontal?
2. What is the northerly component of a wind blowing at 15ms^{-1} from the South East?
3. What is the vector whose components are 3N horizontally and 4N vertically?
4. What are the vertical and horizontal components of a force of 25N acting at 30 º to the horizontal?
5. A plane taking off leaves the runway at 32º to the horizontal, travelling at 180kmh^{-1}. How long will it take to climb to an altitude of 1000m?

Answers

1. 5.5N 2. 11ms^{-1} 3. 5N 50º above the horizontal. 4. 12N vertical, 22N horizontal. 5. 37.8s