ET1.7: TRANSPOSITION WITH CHALLENGES

Possible complications

(i) *The subject variable appears more than once in the formula!*

For example: Make 'm' the subject of $E = m g h + \frac{1}{2} m v^2$

Suggested procedure:

> *Move all terms containing 'm' to one side of the formula and factorize.*

Examples

1. $E = m g h + \frac{1}{2} m v^2$ [All 'm's are already on one side of the formula]

$E = m(g h + \frac{1}{2} v^2)$ [Factorize]

$\frac{E}{gh+\frac{1}{2}v^2} = m$ [÷ (gh + ½v²)]

or $m = \frac{E}{gh+\frac{1}{2}v^2}$

2. $Ir = E - IR$

$Ir + IR = E$ [move 'I's to one side of ormula]

$I(r + R) = E$ [factorize]

$I = \frac{E}{r+R}$ [÷ (r + R)]

(ii) *The formula contains lots of fractions!!*

For example: Make 'u' the subject of $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

Suggested procedure:

> *Cross multiply if possible*
> (ie if there is only one fraction on each side of equation)
>
> otherwise
>
> *Eliminate fractions by multiplying both sides by the lowest common denominator.*

3. *Express in terms of 'u'* $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$ [LCD is *fuv*]

$$fuv.\frac{1}{f} = fuv.\frac{1}{v} - fuv.\frac{1}{u}$$ [multiply all terms by LCD (*fuv*)]

$$uv = fu - fv$$ [cancel and simplify]

$$uv - fu = -fv$$ [move all terms containing 'u' to one side]

$$u(v-f) = -fv$$ [factorize]

$$u = \frac{-fv}{v-f}$$ [÷ (*v – f*)]

4. *Transpose* $m = \sqrt{\frac{d-s}{s(e-f)}}$ *to make 's' the subject.*

$$m = \sqrt{\frac{d-s}{s(e-f)}}$$ [The radical sign acts as a bracket and is removed first].

$$m^2 = \frac{d-s}{s(e-f)}$$ [Square both sides]

$m^2s(e-f)=1(d-s)$ [cross multiply ($m^2 = \frac{m^2}{1}$)]

$m^2se - m^2sf = d - s$ [simplify]

$m^2se - m^2sf + s = d$ [shift all terms with 's' to one side]

$s(m^2e - m^2f + 1) = d$ [factorize]

$$s = \frac{d}{m^2e - m^2f + 1}$$ [÷ ($m^2e - m^2f + 1$)]

(iii) ***the subject is an exponent!!!***

For example: Make 't' the subject of $Q = A \times 10^{kt}$

Suggested

Use logarithms
Remember $\log x^n = n \log x$

procedure:

5. *Express in terms of 't'* $Q = A \times 10^{kt}$

then $\frac{Q}{A} = 10^{kt}$ [÷ 'A' both s]

$\log\left(\frac{Q}{A}\right) = \log 10^{kt}$ [take logs both sides]

$\log\left(\frac{Q}{A}\right) = kt \log 10$ l[og $x^n = n\log x$]

$\log\left(\frac{Q}{A}\right) = kt$ [log 10 = 1]

$\frac{\log\left(\frac{Q}{A}\right)}{k} = t$ or $t = \frac{1}{k}\log\left(\frac{Q}{A}\right)$ [÷ k both sides]

6. *Make 'n' the subject of the formula* $S = P(1+i)^n$

$S = P(1+i)^n$

$\frac{S}{P} = (1+i)^n$ [÷ P both sides]

$\log\left(\frac{S}{P}\right) = \log(1+i)^n$ [take logs both sides]

$\log\left(\frac{S}{P}\right) = n\log(1+i)$ [$\log x^n = n\log x$]

$n = \frac{\log\left(\frac{S}{P}\right)}{\log(1+i)}$ [÷ log (1 + i)]

Sometimes it is helpful to change a logarithmic equation into its equivalent exponential form:

$y = \log_a x \Leftrightarrow a^y = x$

7. *Transform the formula* $T = \frac{1}{c}\log_e(m-A)$ *to make 'm' the subject*

$T = \frac{1}{c}\log_e(m-A)$

$cT = \log_e(m-A)$ [×c both sides]

$e^{cT} = m-A$ [change to exponential equation]

$m = e^{cT} + A$ [+ A both sides]

Exercises

Transpose the following formulae to make the variable in brackets the subject.

1. $M = 10.5C + 35.2(W - \frac{C}{8})$ [C]
2. $At = M(P + t)$ [t]
3. $I = \frac{E}{R}$ [R]
4. $\frac{P}{Q} = \frac{R}{S}$ [S]
5. $I = \frac{E}{R+r}$ [r]
6. $W = \frac{2PR}{R-r}$ [R]
7. $A = \sqrt{\frac{2q(L-r)}{rL}}$ [L]
8. $E = \frac{w^2 a}{(w^2+m)b^3}$ [w]
9. $H = Ae^{-kt}$ [t]
10. $\frac{1}{q^2}\log_e\left(\frac{M}{2}\right) = P$ [M]

Answers (NB: there may be alternative answers that are algebraically equivalent)

1. $C = \frac{M-35.2W}{6.1}$
2. $t = \frac{MP}{A-M}$
3. $R = \frac{E}{I}$
4. $S = \frac{RQ}{P}$
5. $r = \frac{E-IR}{I}$
6. $R = \frac{wr}{w-2P}$
7. $L = \frac{2qr}{2q-A^2r}$
8. $w = \pm\sqrt{\frac{Eb^3m}{a-Eb^3}}$
9. $-\frac{1}{k}\log_e\left(\frac{H}{A}\right)$
10. $M = 2e^{pq^2}$