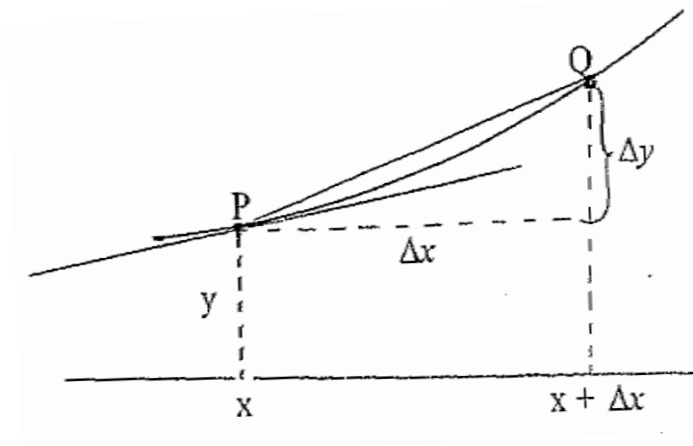


## Applications of Differentiation

# DN1.11: SMALL CHANGES AND APPROXIMATIONS

Consider a function defined by  $y = f(x)$ .



If  $x$  is increased by a small amount  $\Delta x$  to  $x + \Delta x$ ,

then as  $\Delta x \rightarrow 0$ ,  $\frac{\Delta y}{\Delta x} \rightarrow \frac{dy}{dx}$  and  $\frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$

Therefore if  $\Delta x$  is small,

$$\Delta y \approx \frac{dy}{dx} \Delta x \quad \text{or} \quad \Delta y \approx f'(x) \Delta x$$

This result can be used find solutions to problems of the type:

- If the radius of a circle is increased by 3% what will be the corresponding change in the area?
- If the radius of a circle is measured to be  $10.0 \pm 0.2$  what will be the maximum error in calculating the area?

## Examples

1. *The side of a square is 5cm. How much will the area of the square increase when the side expands by 0.01cm?*

Let the area of the square be  $A$  and the length of a side be  $x$  cm.

$$\text{Then } A = x^2 \quad \text{and} \quad \frac{dA}{dx} = 2x$$

$$\Delta x \approx \frac{dA}{dx} \Delta x$$

$$= 2x \Delta x$$

$$= 2 \times 5 \times 0.01$$

$$= 0.1$$

The increase in area  $\approx 0.1 \text{ cm}^2$

2. *A 2% error is made in measuring the radius of a sphere. Find the percentage error in the volume.*

Let the radius be  $r$  and the volume be  $V$

$$\text{Then } V = \frac{4}{3}\pi r^3 \quad \text{and} \quad \frac{dV}{dr} = 4\pi r^2$$

$$\Delta V \approx \frac{dV}{dr} \Delta r$$

$$= 4\pi r^2 \Delta r$$

$$= 4\pi r^2 \cdot 0.02r$$

$$= 0.08\pi r^3$$

The percentage error in the volume

$$\approx \frac{\Delta V}{V} \times 100 \%$$

$$= \frac{0.08\pi r^3}{\frac{4}{3}\pi r^3} \times 100 \%$$

$$= \frac{0.08}{\frac{4}{3}} \times 100 \%$$

$$= 6 \%$$

The percentage error in the volume  $\approx 6\%$

## Exercises

1. If the radius of a sphere is increased from 10cm to 10.1 cm what is the approximate increase in surface area?
2. The height of a cylinder is 10 cm and its radius is 4cm. Find the approximate increase in volume when the radius increases to 4.02 cm.
3. An error of 3% is made in measuring the radius of a sphere. Find the percentage error in volume.
4. An error of  $2\frac{1}{2}\%$  is made in the measurement of the area of a circle. What is the percentage error in
  - a) the radius?
  - b) the circumference?
5. One side of a rectangle is three times the other. If the perimeter increases by 2% what is the percentage increase in area?
6. The kinetic energy  $K$  of a body of mass  $m$  moving with speed  $v$  is given by  $K = \frac{1}{2}mv^2$ . If a body's speed is increased by 1.5% what is the approximate percentage change in the kinetic energy?

## Answers

- 1)  $25.13\text{cm}^2$     2)  $5.03\text{cm}^3$     3) 9%    4)a. 1.25%    b. 1.25%    5) 4%    6) 3%