ET1.4: SIMULTANEOUS EQUATIONS

Equations with Two Variables

Two equations in two variables are said to be *simultaneous* if both must be considered at the same time.

An *ordered pair* (x, y) which satisfies **both** equations is said to be a solution for the simultaneous equations.

Two methods for solving a pair of simultaneous equations using algebra are

- elimination
- substitution

Elimination Method

In the *elimination method* we manipulate the equations so that one of the variables has coefficients that are equal and opposite in sign. Then we add the equations to eliminate that variable.

Examples

1. *Solve the simultaneous equations* $x + y = 6$ *and* $x - y = 2$

$x + y = 6$ [1] [Label the equations]
$x - y = 2$ [2] [NB: The 'y' coefficients are equal and opposite]

$2x = 8$ [1] + [2] [The equations are added eliminating y]
$x = 4$

Substitute x = 4 back into equation [1] or [2]:

$4 + y = 6$ Using [1]
$y = 2$

The solution is (4,2) [We can check the solution in the original equations
$4 + 2 = 6$ and $4 - 2 = 2$!]

2. *Solve simultaneously* $3x + 2y = 10$ *and* $4x + 3y = 13$

$3x + 2y = 10$ [1] [Label the equations]
$4x + 3y = 13$ [2] [NB: The equations must be manipulated so that one variable has equal and opposite coefficients]

$12x + 8y = 40$ [1]' = [1] x 4 [Equation [1] has been multiplied by 4]
$-12x - 9y = -39$ [2]' = [2] x -3 [Equation [2] has been multiplied by -3]

$-y = 1$ [1]' + [2]' [Equations [1]' and [2]' have been added]
$y = -1$

Substituting y = -1 in [1]:
$3x + 2(-1) = 10$
$3x = 12$
$x = 4$
The solution is (4,-1) [Check $3(4) + 2(-1) = 10$ and $4(4) + 3(-1) = 13$]

Substitution Method

In the *substitution method* the simpler equation is transposed to make one of the variables the subject, and then substitution enables the second equation to be reduced to one variable.

Examples

1. *Solve the simultaneous equations*

$x + y = 6$ *[1]*
and $x - y = 2$ *[2]* [It is convenient to label the equations]

From [1] $x + y = 6 \Rightarrow y = 6 - x$

Substitute for y in [2]: $x - (6 - x) = 2$

$x - 6 + x = 2$

$2x - 6 = 2$

$2x = 8$

$x = 4$

and from [1] $y = 6 - 4$

$= 2$

The solution is (4,2) which can be checked as before

2. *Solve* $3x + 2y = 10$ *[1]*
and $4x + 3y = 13$ *[2]*

From [1] $3x = 10 - 2y \Rightarrow x = \dfrac{10 - 2y}{3}$

Substitute in [2]: $4\left(\dfrac{10 - 2y}{3}\right) + 3y = 13$ [multiply each side by 3 to eliminate fractions]

$4(10 - 2y) + 9y = 39$

$40 - 8y + 9y = 39$

$40 + y = 39$

$y = -1$

and from [1]: $x = \dfrac{10 - 2(-1)}{3}$

$= 4$

The solution is (4,-1)

NB: Some pairs of simultaneous equations have no solution and some have multiple solutions. For example attempt to solve

(i) $2x - y = 6$ and $-4x + 2y = -7$

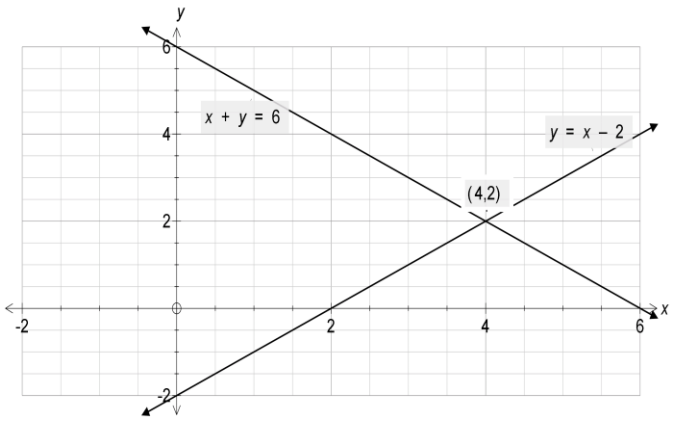
and (ii) $y = 1 - 3x$ and $6x = 2(y - 1)$

Graphical Method

In the *graphical method* the straight line graphs representing both equations are plotted on the same set of axes. The solution is the point of intersection of the two lines.
(the graphs must be plotted accurately for this method to give an accurate answer)

Example

Solve the simultaneous equations $x + y = 6$ *and* $x - y = 2$ *graphically*



[Graphs representing each linear equation are drawn]

[Point of intersection is read from the graph]

The solution is (4,2)

Check by substituting in each equation: 4 + 2 = 6 *and* 4 – 2 = 2

Exercises

Exercise 1

Solve simultaneously using either the elimination or substitution method:

1. y = x – 3 and 5x – 2y = 18
2. y = 3x + 4 and 2x + 3y = 23
3. 4x + y = 23 and x – y = 2
4. -3x + 2y = -4 and 5x – 2y = 8
5. 3x – 5y = 14 and 2x + y = 5
6. 3x + 2y = 18 and 2x – 5y = -7

Exercise 2

Solve the following simultaneous equations graphically.

1. $y = x - 3$ and $x + y = 5$
2. $5x - y = 8$ and $2y - 3x = -2$
3. $x + y = 1$ and $y - 2x = 7$

Answers

Exercise 1

1. (4,1) 2. (1,7) 3. (5,3) 4. (2,1) 5. (3,-1) 6. (4,3)

Exercise 2

1. (4,1) 2. (2,2) 3. (-2,3)