

DN1.4: RULES FOR DIFFERENTIATION

Operational Rules

The following rules for differentiation can be established very easily from 'first principles':

1. If $g(x) = k f(x)$, where k is a constant

$$\text{then } g'(x) = k f'(x)$$

2. If $f(x) = k$ where k is a constant

$$\text{then } f'(x) = 0$$

3. If $f(x) = g(x) + h(x)$

$$\text{then } f'(x) = g'(x) + h'(x)$$

Derivative of a power of x

$$\text{If } y = x^n, \text{ then } \frac{dy}{dx} = nx^{n-1}$$

Examples

1. If $y = x^7$, then $\frac{dy}{dx} = 7x^6$

2. If $y = x^{2011}$, then $\frac{dy}{dx} = 2011x^{2010}$

3. If $y = \sqrt{x} = x^{\frac{1}{2}}$, [Rewrite the expression in index form before differentiating]

$$\text{then } \frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$4. \text{ If } y = \frac{1}{x^2} = x^{-2}, \text{ then } \frac{dy}{dx} = -2x^{-3} = \frac{-2}{x^3}$$

And using some of the operational rules:

$$5. \ y = x^3 + 7x^6, \quad \frac{dy}{dx} = 3x^2 + 42x^5$$

$$6. \ y = \frac{4}{x} + 10 = 4x^{-1} + 10, \quad \frac{dy}{dx} = -4x^{-2} = \frac{-4}{x^2}$$

$$7. \ y = \frac{x^4 - 1}{\sqrt{x}} = x^{\frac{7}{2}} - x^{\frac{-1}{2}}, \quad [\text{divide through by } x^{\frac{1}{2}}]$$

$$\frac{dy}{dx} = \frac{7}{2}x^{\frac{5}{2}} + \frac{1}{2}x^{\frac{-3}{2}}$$

Derivatives of Other Functions

Function	Derivative
$f(x) = e^x$	$f'(x) = e^x$
$f(x) = \log_e x$	$f'(x) = \frac{1}{x}$
$f(x) = \sin(x)$	$f'(x) = \cos(x)$
$f(x) = \cos(x)$	$f'(x) = -\sin(x)$
$f(x) = \tan(x)$	$f'(x) = \sec^2 x$

Examples

$$1. \ y = \sin(x) + 3x^2, \quad \frac{dy}{dx} = \cos(x) + 6x$$

$$2. \ y = e^x - 5, \quad \frac{dy}{dx} = e^x$$

$$3. \ y = 10\left(\cos x - \frac{1}{2x}\right) = 10\cos x - \frac{5}{x}, \quad \frac{dy}{dx} = -10\sin x + \frac{5}{x^2}$$

Exercises

1. Differentiate the following

a) x^7 b) $x^{\frac{1}{5}}$ c) x^{-19} d) $\frac{1}{x^4}$ e) 53
f) $\sqrt[4]{x}$ g) $5x^6$ h) $9x^{-5}$ i) $\frac{\sqrt{5}}{2x^4}$ j) $3x^2 + 2x$

2. Find the derivative of

a) $\sin x - \cos x$ b) $10 - \log_e x$
c) $\tan x - \sqrt{x}$ d) $3\sin(x) - 2 + \frac{1}{x}$
e) $\frac{e^x}{6} - x^7$

Answers

1. a) $7x^6$ b) $\frac{1}{5x^5}$ c) $-19x^{-20}$ d) $-\frac{4}{x^5}$ e) 0
f) $\frac{1}{4x^4}$ g) $30x^5$ h) $-45x^{-6}$ i) $-\frac{2\sqrt{5}}{x^5}$ j) $6x + 2$

2. a) $\cos x + \sin x$ b) $-\frac{1}{x}$ c) $\sec^2 x - \frac{1}{2\sqrt{x}}$ d) $3\cos x - \frac{1}{x^2}$
e) $\frac{e^x}{6} - 7x^6$