RMIT UNIVERSITY

DN1.7: QUOTIENT RULE

The **quotient rule** is used when we want to differentiate a function which is the quotient of two simpler functions:

$$\text{If} \quad f(x) = \frac{u(x)}{v(x)} \quad \text{then} \quad f'(x) = \frac{v(x) \,.\, u'(x) - u(x) \,.\, v'(x)}{[v(x)]^2}$$

which is often abbreviated to

$$y' = \frac{v\,u' - u\,v'}{v^2}$$

Examples

1) *If* $y = \dfrac{1+x}{x^2-3}$ *find* $\dfrac{dy}{dx}$.

$u = 1 + x \Rightarrow u' = 1$ and $v = x^2 - 3 \Rightarrow v' = 2x$

$$y' = \frac{v\,u' - u\,v'}{v^2}$$

$$= \frac{(x^2-3)1-(1+x)2x}{(x^2-3)^2} \qquad \text{and simplify if possible…}$$

$$= \frac{x^2 - 3 - 2x - 2x^2}{(x^2-3)^2}$$

$$= \frac{-x^2 - 2x - 3}{(x^2-3)^2}$$

2) *Differentiate* $\dfrac{x^2}{\log_e x}$

$u = x^2$, $u' = 2x$ and $v = \log_e x$ and $v' = \dfrac{1}{x}$

$$y' = \frac{v\,u' - u\,v'}{v^2}$$

$$= \frac{\log_e x \,.\, 2x - x^2 \frac{1}{x}}{(\log_e x)^2}$$

$$= \frac{2x\log_e x - x}{(\log_e x)^2} \qquad \text{after simplifying}$$

Exercise

Find the derivatives of the following functions

1) f(x) = $\frac{2x+1}{4x-3}$

2) f(x) = $\frac{3}{3x^2+1}$

3) y = $\frac{x^3+4}{3x^3-2}$

4) f(x) = $\frac{-1}{x^2+\sqrt[3]{x}}$

5) y = $\frac{\sqrt{x}}{1-\sqrt{x}}$

6) y = $\frac{1}{(x^2-1)^4}$

7) y = $\frac{\tan x}{x}$

8) y = $\frac{e^{2x}}{\sin^2 x}$

Answers

1) $\frac{-10}{(4x-3)^2}$

2) $\frac{-18x}{(3x^2+1)^2}$

3) $\frac{-42x^2}{(3x^3-2)^2}$

4) $\frac{2x+\frac{1}{3}x^{\frac{-2}{3}}}{(x^2+x^{\frac{1}{3}})^2}$

5) $\frac{1}{2x^{\frac{1}{2}}(1-x^{\frac{1}{2}})^2}$ after simplifying!

6) $\frac{-8x}{(x^2-1)^5}$

7) $\frac{x\sec^2 x-\tan x}{x^2}$

8) $\frac{2e^{2x}\left(\sin x-\cos x\right)}{\sin^3 x}$ after simplifying !