RMIT UNIVERSITY

ET1.3: QUADRATIC FORMULA

The solutions to any quadratic equation $ax^2 + bx + c = 0$ can be found by substituting the values a, b, c into the *quadratic formula:*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples

1. $x^2 - 5x + 4 = 0 \quad \rightarrow \quad a = 1, \quad b = -5, \quad c = 4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{5 \pm \sqrt{25 - 4(1)(4)}}{2}$$

$$x = \frac{5 \pm \sqrt{9}}{2}$$

$$x = \frac{5 + \sqrt{9}}{2} \quad \text{or} \quad x = \frac{5 - \sqrt{9}}{2}$$

$x = 4$ or $x = 1$

2. $x^2 + x + 10 = 0 \quad \rightarrow \quad a = 1, \quad b = 1, \quad c = 10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(10)}}{2}$$

$$x = \frac{-1 \pm \sqrt{-39}}{2}$$ which has no solution

Discriminant

NB: The part of the quadratic formula which is under the radical sign $b^2 - 4ac$ is called the *discriminant*. Its value determines the number of solutions and whether they will be rational or irrational.

$b^2 - 4ac < 0 \rightarrow$ no solutions

$b^2 - 4ac = 0 \rightarrow$ one solution

$b^2 - 4ac > 0 \rightarrow$ two solutions

If $b^2 - 4ac$ is a perfect square the solutions will be rational.

Exercise

Solve the following quadratic equations (if possible)

1. $x^2 - 4x - 7 = 0$
2. $x^2 - 2x - 2 = 0$
3. $x^2 + 6x - 9 = 0$
4. $x^2 - x - 7 = 0$
5. $4x^2 - 12x + 6 = 0$
6. $2x^2 + x - 2 = 0$
7. $x^2 - 4x + 2 = 0$
8. $2x^2 - 3x + 2 = 0$
9. $3x^2 + 5x - 7 = 0$
10. $3x^2 = x + 1$
11. $4x^2 + x + 3 = 0$
12. $\frac{3x+1}{2} = \frac{x+1}{x}$

Answers

1. $x = \frac{4 \pm \sqrt{44}}{2}$ $(2 \pm \sqrt{11})$
2. $x = \frac{2 \pm \sqrt{12}}{2}$ $(1 \pm \sqrt{3})$
3. $x = \frac{-6 \pm \sqrt{72}}{2}$ $(-3 \pm 3\sqrt{2})$
4. $x = \frac{1 \pm \sqrt{29}}{2}$
5. $x = \frac{12 \pm \sqrt{48}}{8}$ $(\frac{3 \pm \sqrt{3}}{2})$
6. $x = \frac{-1 \pm \sqrt{17}}{4}$
7. $x = \frac{4 \pm \sqrt{8}}{2}$ $(2 \pm \sqrt{2})$
8. no solution
9. $x = \frac{-5 \pm \sqrt{109}}{6}$
10. $x = \frac{1 \pm \sqrt{13}}{6}$
11. no solution
12. $x = -\frac{2}{3}$, $x = 1$