

# ET1.2: QUADRATIC EQUATIONS

## General form

A *quadratic* equation can be rearranged to the form

$$ax^2 + bx + c = 0 \quad a \neq 0$$

### Examples:

$$1. \quad 5x^2 - 3x + 9 = 0 \qquad a = 5, \quad b = -3, \quad c = 9$$

$$2. \quad x^2 = 5x - 4 \Rightarrow x^2 - 5x + 4 = 0 \qquad a = 1, \quad b = -5, \quad c = 4$$

$$3. \quad x = \frac{3}{2x} \Rightarrow 2x^2 = 3$$

$$\qquad \qquad \qquad \Rightarrow 2x^2 - 3 = 0 \qquad a = 1, \quad b = 0, \quad c = -3$$

## Factorisation

If the equation can be factorised then the 'null factor law' can be used to find the solutions:

Null factor law:  
If  $m \times n = 0$ , then  $m = 0$  and/or  $n = 0$

If the product of two or more factors is zero then any one of the individual factors may be zero and provide a solution for the equation.

### Examples:

$$1. \quad y^2 = 5y$$

$$y^2 - 5y = 0 \qquad \text{[rearrange to form } ax^2 + bx + c = 0]$$

$$y(y - 5) = 0 \qquad \text{[factorise]}$$

$$y = 0 \quad \text{or} \quad y - 5 = 0 \qquad \text{[null factor law]}$$

$$\therefore y = 0 \quad \text{or} \quad y = 5$$

$$y = 0: 0^2 = 5 \times 0 \quad y = 5: 5^2 = 5 \times 5 \qquad \text{[check by substitution]}$$

$$2. \quad x^2 - 5x + 4 = 0$$

$$(x-4)(x-1) = 0 \quad \text{[factorise]}$$

$$x-4 = 0 \quad \text{or} \quad x-1 = 0 \quad \text{[null factor law]}$$

$$x = 4 \quad \text{or} \quad x = 1 \quad \text{[check by substitution]}$$

$$x = 4: 4^2 - 5 \times 4 + 4 = 0$$

$$x = 1: 1^2 - 5 \times 1 + 4 = 0$$

$$3. \quad p^2 + 10p + 25 = 0$$

$$(p+5)(p+5) = 0 \quad \text{[factorise]}$$

$$p+5 = 0 \quad \text{[null factor law]}$$

$$p = -5 \quad \text{check: } (-5)^2 + 10 \times (-5) + 25 = 0$$

$$4. \quad 4m^2 - 49 = 0$$

$$(2m+7)(2m-7) = 0 \quad \text{[factorise by difference of squares]}$$

$$2m+7 = 0 \quad \text{or} \quad 2m-7 = 0 \quad \text{[null factor law]}$$

$$m = \frac{-7}{2} \quad \text{or} \quad m = \frac{7}{2}$$

$$5. \quad x = \frac{-6}{1-2x} \quad \text{provided } x \neq 1/2$$

$$x - 2x^2 = -6$$

$$2x^2 - x - 6 = 0 \quad \text{[rearrange to form } ax^2 + bx + c = 0]$$

$$(2x+3)(x-2) = 0 \quad \text{[factorise]}$$

$$x = -\frac{3}{2} \quad \text{or} \quad x = 2 \quad \text{[solve using null factor law]}$$

## Exercise

Solve the following quadratic equations.

$$(1) \quad x^2 - 6x + 8 = 0$$

$$(2) \quad x^2 + 2x - 3 = 0$$

$$(3) \quad 2x^2 - 3x - 2 = 0$$

$$(4) \quad 6 - z - z^2 = 0$$

$$(5) \quad 2x^2 + 7x = 15$$

$$(6) \quad 11p = 3(2p^2 + 1)$$

### Answers

$$(1) \quad x = 4, x = 2$$

$$(2) \quad x = -3, x = 1$$

$$(3) \quad x = -\frac{1}{2}, x = 2$$

$$(4) \quad z = -3, z = 2$$

$$(5) \quad x = \frac{3}{2}, x = -5$$

$$(6) \quad p = \frac{1}{3}, p = \frac{3}{2}$$