

# DN1.6: PRODUCT RULE

The 'product rule' is used when we want to differentiate the product of two functions

$$\text{If } f(x) = u(x).v(x)$$

$$\text{then } f'(x) = u(x).v'(x) + u'(x).v(x)$$

which is often abbreviated to

$$y' = uv' + u'v$$

## Examples

1. Find the derivative of  $f(x) = (x + 3)^6 (2x - 1)$

$$\text{Let } u = (x+3)^6 \quad \text{and} \quad \text{let } v = 2x - 1$$

$$u' = 6(x+3)^5 \quad v' = 2$$

$$\text{Then } y' = uv' + u'v$$

$$= (x + 3)^6 .2 + 6(x + 3)^5 (2x - 1)$$

$$= 2(x + 3)^6 + 6(x + 3)^5 (2x - 1)$$

...and this could (but does not have to be) simplified further.....

$$= 2(x + 3)^5 [(x + 3) + 3(2x - 1)] \quad [\text{by factorizing}]$$

$$= 2(x + 3)^5 (7x)$$

$$= 14x(x + 3)^5$$

2. Differentiate  $e^x \sin 2x$

$$\text{Let } u = e^x \quad \text{and} \quad v = \sin 2x$$

$$u' = e^x \quad \text{and} \quad v' = 2\cos 2x \quad [\text{using the chain rule}]$$

$$\text{Then } y' = u v' + u' v$$

$$= e^x .2\cos 2x + e^x \sin 2x$$

$$= 2e^x \cos 2x + e^x \sin 2x$$

## Exercises

1. Use the product rule to differentiate the following

a)  $y = (x - 2)(6x + 7)$       b)  $f(x) = (2x^2 + 4)(x^5 + 4x^2 - 2)$   
[simplify as far as possible]

c)  $y = (\sqrt{x} - 1)(x^2 + 1)$       d)  $y = (x^3 - 4x + \sqrt{x})(3x^4 + 2)$   
[simplify as far as possible]

e)  $f(x) = \sqrt{x-1}(x+1)^2$       f)  $f(x) = (x^2 - 1)(x^2 + 1)$   
[simplify as far as possible]

2. Find the derivative of

a)  $y = e^x \tan x$       b)  $y = x^2 \log_e(x^2)$

c)  $y = \sin x \cos x$       d)  $y = x \sin x$

e)  $y = (2 - x)\tan 3x$       f)  $y = \frac{e^x}{x}$  [Hint:  $\frac{1}{x} = x^{-1}$ ]

## Answers

1. (a)  $12x - 5$   
(b)  $(2x^2 + 4)(5x^4 + 8x) + 4x(x^5 + 4x^2 - 2)$   
(c)  $\frac{5}{2}x^{\frac{3}{2}} - 2x + \frac{1}{2x^2}$   
(d)  $(x^3 - 4x + x^{\frac{1}{2}})(12x^3) + (3x^2 - 4 + \frac{1}{2x^{\frac{1}{2}}})(3x^4 + 2)$   
(e)  $2(x+1)\sqrt{x-1} + \frac{(x+1)^2}{2\sqrt{x-1}}$   
(f)  $4x^3$

2. (a)  $e^x \tan x + e^x \sec^2 x$   
(b)  $2x + 2x \log_e(x^2)$   
(c)  $\cos^2 x - \sin^2 x$   
(d)  $x \cos x + \sin x$   
(e)  $3(2-x)\sec^2 3x - \tan 3x$   
(f)  $\frac{e^x}{x} - \frac{e^x}{x^2}$