RMIT UNIVERSITY

AS1.4: POLYNOMIAL LONG DIVISION

One polynomial may be divided by another of lower degree by long division (similar to arithmetic long division).

Example

$$\frac{(x^3+3x^2+x+9)}{(x+2)}$$

1. Write the question in long division form.

$$x+2\overline{\smash{)}x^3+3x^2+x+9}$$

2. Begin with the x^3 term.
   x^3 divided by x equals x^2.
   Place x^2 above the division bracket as shown.

$$\begin{array}{rl} & x^2 \\ x+2 & \overline{\smash{)}x^3+3x^2+x+9} \\ \text{subtract} & x^3+2x^2 \\ & \quad\quad x^2 \end{array}$$

3. Multiply $x+2$ by x^2.
   $x^2(x+2)=x^3+2x^2$.
   Place x^3+2x^2 below x^3+3x^2 then subtract.
   $x^3+3x^2-(x^2+2x)=x^2$

4. "Bring down" the next term (x) as indicated by the arrow.

Repeat this process with the result until there are no terms remaining.

$$\begin{array}{rl} & x^2+x \\ x+2 & \overline{\smash{)}x^3+3x^2+x+9} \\ & x^3+2x^2 \quad \downarrow \\ & \quad\quad \underline{x^2+x} \end{array}$$

5. x^2 divided by x equals x.
   Multiply
   $x(x+2)=x^2+2x$.
   $(x^2+x)-(x^2+2x)=-x$

6. Bring down the 9.

7. $-x$ divided by x equals -1.
   Multiply
   $-1(x+2)=-x-2$.
   $(-x+9)-(-x-2)=11$

$$\begin{array}{rl} & x^2+x-1 \\ x+2 & \overline{\smash{)}x^3+3x^2+x+9} \\ & x^3+2x^2 \quad \downarrow \quad \downarrow \\ & \quad\quad \underline{x^2+x} \quad \downarrow \\ & \quad\quad x^2+2x \quad \downarrow \\ & \quad\quad\quad \underline{-x+9} \\ & \quad\quad\quad -x-2 \\ & \quad\quad\quad\quad \underline{+11} \end{array}$$

The remainder is 11

$\frac{(x^3+3x^2+x+9)}{(x+2)}$ equals x^2+x-1, with remainder 11.

If the polynomial has an 'x^n' term missing, add the term with a coefficient of zero.

Example

$\left(2x^3-3x+1\right)\div\left(x-1\right)$

Rewrite $\left(2x^3-3x+1\right)$ as $\left(2x^3+0x^2-3x+1\right)$

Divide using the method from the previous example.

$$2x^3 \div x = 2x^2$$
$$2x^2\left(x-1\right)=2x^3-2x^2$$
$$2x^3-0x^2-\left(2x^3-2x^2\right)=2x^2$$

$$2x^2 \div x = 2x$$
$$2x\left(x-1\right)=2x^2-2x$$
$$2x^2-3x-\left(2x^2-2x\right)=-x$$

$$-x\div x=-1$$
$$-1\left(x-1\right)=-x+1$$
$$-x+1-\left(x+1\right)=0$$

Remainder is 0

$$
\begin{array}{rl}
 & 2x^2+2x-1 \\
x-1\,\big) & 2x^3+0x^2-3x+1 \\
 & 2x^3-\ 2x^2 \quad \downarrow \quad \downarrow \\
 & \underline{2x^2-3x} \quad \downarrow \\
 & 2x^2-\ 2x \quad \downarrow \\
 & \underline{-x+1} \\
 & -x+1 \\
 & \underline{0}
\end{array}
$$

$\left(2x^3-3x+1\right)\div\left(x-1\right)=\left(2x^2+2x-1\right)$ with remainder = 0

$\therefore\left(2x^3-3x+1\right)=\left(x-1\right)\left(2x^2+2x-1\right)$

See Exercise 1.

Polynomial long division can be used help factorise cubic (and higher order) equations.

Solving Cubic Equations

Factor Theorem

If P(x) is a polynomial in x and

P(a) = 0 then (x – a) is a factor of P(x)

We can use the factor theorem to find one factor of a cubic function, and then use polynomial long division to find the remaining factor(s).
(Sometimes it is possible to find all solutions by finding three values of x for which P(x) = 0).
A cubic equation has a maximum of three distinct solutions.

Example

Solve the equation $x^3 - 5x^2 - 2x + 24 = 0$

Look for a value of *x* that makes P(*x*) = 0.

Try *x* = 1 first, then work up through the factors of 24.

Try x = 1 P(1) =18
x – 1 is not a factor

Try x = 2 P(2) = 8
x – 2 is not a factor

Try x = – 2 P(–2) = 0
P(–2) = 0 therefore
x + 2 is a factor

Next divide $x^3 - 5x^2 - 2x + 24$ by (*x* + 2), using the method of polynomial long division shown above.
The remainder should be zero

$$
\begin{array}{r}
x^2 - 7x + 12 \\
x + 2 \overline{)\, x^3 - 5x^2 - 2x + 24} \\
x^3 + 2x^2 \\
\underline{-7x^2 - 2x} \\
-7x^2 - 14x \\
\underline{12x + 24} \\
12x + 24 \\
\underline{0}
\end{array}
$$

The factors of $x^3 - 5x^2 - 2x + 24$ are (x + 2) and (x2 – 7x + 12)
The factors of $x^2 - 7x + 12$ are (x – 3) and (x – 4).

$x^3 - 5x^2 - 2x + 24 = 0$ has solutions x = – 2, x = 3 and x = 4

Exercises

Exercise 1

Divide the first polynomial by the second.

(a) $2x^3 - 6x^2 + 5x + 2,\ \ x - 2$

(b) $3x^3 + 13x^2 + 6x - 12,\ \ 3x + 4$,

(c) $3x^3 + x - 1,\ \ x + 1$

(d) $x^3 + 2x - 3,\ \ x - 1$

(e) $2x^4 + 5x^3 + x + 3,\ \ 2x + 1$

(f) $2x^3 + x^2 + 5x + 12,\ \ 2x + 3$

Exercise 2

Solve the following equations.

(a) $x^3 + 7x^2 + 11x + 5 = 0$

(b) $4x^3 + 2x^2 - 2x = 0$

(c) $-x^3 - 3x^2 + x + 3 = 0$

(d) $x^3 - 7x - 6 = 0$

Answers

Exercise 1

(a) $2x^2 - 2x + 1$ remainder 4

(b) $x^2 + 3x - 2$ remainder -4

(c) $3x^2 - 3x + 4$ remainder – 3

(d) $x^2 + x + 3$ remainder 0

(e) $x^3 + 2x^2 - x + 1$ remainder 2

(f) $x^2 - x + 4$ remainder 0

Exercise 2

(a) $x = -5$, $x = -1$

(b) $x = 0$, $x = 1$, $x = -1/2$

(c) $x = 1$, $x = -3$, $x = -1$

(d) $x = -1$, $x = -2$, $x = 3$