

## PMO1.2: LINEAR MOTION: GRAPHS

Linear motion refers to the motion of an object in a straight line.

### Displacement ( $x$ )

When an object is moved from one point to another it is said to be displaced. If we move to Dandenong we can say we have been displaced 32 km from Melbourne, but to completely define our position we must say we are 32 km South East of Melbourne, ie we have given both a magnitude (32 km) and a direction (South East). Displacement is a vector quantity.

**Note:** Displacement is sometimes referred to as 'position'. See exercises at the end of this worksheet.

### Distance ( $d$ )

Distance is the magnitude (size) of displacement, but has no direction so it is a **scalar** quantity, ie my distance is 60 km, but my displacement is 60 km due West.

### Velocity ( $v$ )

Velocity is the rate of change of displacement per second, ie

$$v = \frac{x}{t} \quad \text{where } x \text{ is in metres, and } t \text{ is in seconds. Velocity has units of } \text{ms}^{-1}.$$

Since velocity involves displacement it is also a vector quantity.

### Speed ( $v$ )

Speed is the magnitude (size) of velocity, but has no direction so it is a scalar quantity, ie

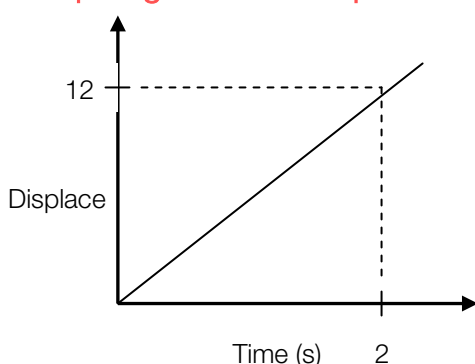
My speed is  $60 \text{ kmh}^{-1}$ , but my velocity is  $60 \text{ kmh}^{-1}$  due East.  
The unit of speed is also  $\text{ms}^{-1}$ .

### Acceleration ( $a$ )

Acceleration is the rate of change of velocity per second, ie.

$$a = \frac{v}{t} \quad \text{where } v \text{ is in } \text{ms}^{-1} \text{ and } t \text{ is in seconds. So acceleration has units of } \text{ms}^{-2}.$$

### Graphing Motion: Displacement-Time graphs



A person runs 12 metres in 2 seconds.

$$v = \frac{x}{t} = \frac{12}{2} = 6 \text{ms}^{-1}$$

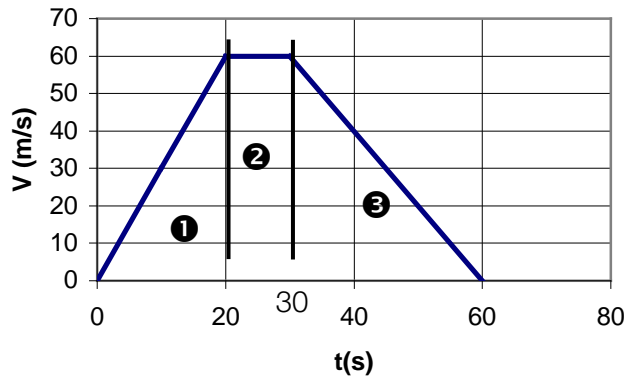
**Note:** the gradient of a Displacement-Time graph gives the velocity

Gradient of distance-time graph = speed

Gradient of displacement-time graph = velocity

### Velocity-Time graphs

The velocity-time graph below shows a car which accelerates uniformly from rest to  $60 \text{ ms}^{-1}$  in 20 seconds, then travels at a constant velocity of  $60 \text{ ms}^{-1}$  for the next 10 seconds, then decelerates uniformly to rest in 30 seconds. The total journey took 60 seconds.



### Area under a velocity-time graph

Area = length  $\times$  width = metres/second  $\times$  seconds = metres.

Area under a velocity-time graph gives the displacement.

Area under a speed-time graph gives the distance.

We can divide the graph into 3 distinct sections and calculate the area for each.

Area ❶ Triangle =  $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 20 \times 60 = 600$  metres.

Area ❷ Rectangle = Length  $\times$  Width =  $60 \times 10 = 600$  metres.

Area ❸ Triangle =  $\frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 30 \times 60 = 900$  metres.

Total displacement = Total area =  $600 + 600 + 900 = 2100$  metres

### Gradient of a velocity-time graph

$$\text{Gradient} = \frac{\text{Rise}}{\text{Run}} = \frac{\text{Change in Velocity}}{\text{Time}} = \text{Acceleration}$$

0-20 seconds: Gradient =  $60/20 = 3 \text{ ms}^{-1}$ . Car is accelerating at  $3 \text{ ms}^{-1}$ .

20-30 seconds: Gradient =  $0/10 = 0 \text{ ms}^{-1}$ .

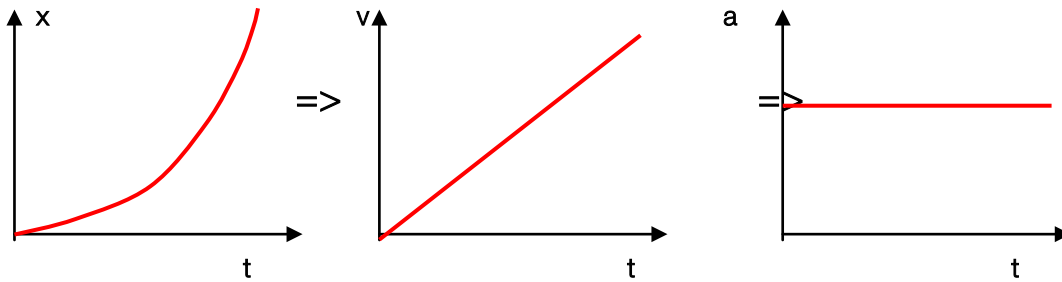
**Note:** No slope  $\therefore$  No acceleration. Car is travelling at a constant velocity of  $60 \text{ m/s}$ .

30-60 seconds. Gradient =  $-60/30 = -2 \text{ ms}^{-1}$

**Note:** Negative slope  $\therefore$  Negative acceleration, or deceleration. Car is slowing down.

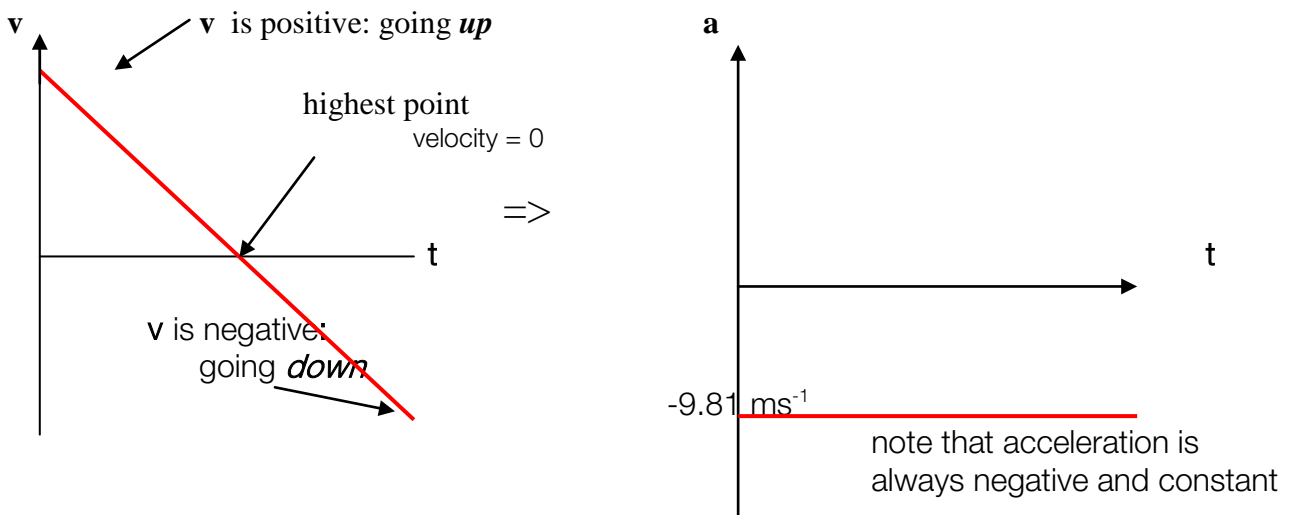
## Some further examples

**Example 1:** A car accelerates from a stationary position. The acceleration is constant (uniform).



Note that the slope of the “ $x$ - $t$ ” graph gives a “ $v$ - $t$ ” graph, and the slope of the “ $v$ - $t$ ” graph gives an “ $a$ - $t$ ” graph.

**Example 2:** A ball is thrown vertically upwards.

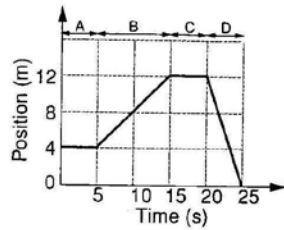


## Summary

| Graph type<br>Found from | $x - t$     | $v - t$      | $a - t$            |
|--------------------------|-------------|--------------|--------------------|
| Gradient                 | Velocity    | Acceleration | Meaningless        |
| Area under graph         | Meaningless | Displacement | Change in velocity |

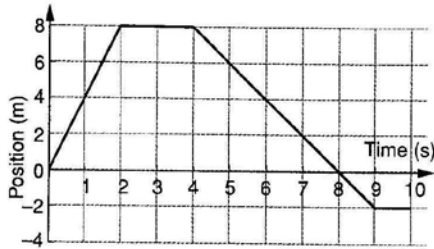
## Exercise

- 1 The graph shows the position of a dancer moving across a stage.



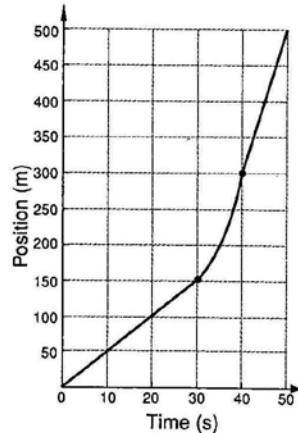
- What was the starting position of the dancer?
- In which of the sections (A–D) is the dancer at rest?
- In which of the sections is the dancer moving in a positive direction?
- In which of the sections is the dancer moving with a negative velocity?

The following information relates to questions 2–6. The graph represents the straight line motion of a radio-controlled toy car.

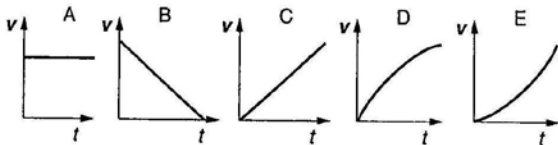


- Describe the motion of the car.
- What was the position of the car after:
  - 2 s?
  - 4 s?
  - 6 s?
  - 10 s?
- When did the car return to its starting point?
- What was the velocity of the car:
  - during the first 2 s?
  - after 3 s?
  - from 4 s to 8 s?
  - at 8 s?
  - from 8 s to 9 s?
- During its 10 s motion, what was the car's:
  - distance travelled?
  - displacement?

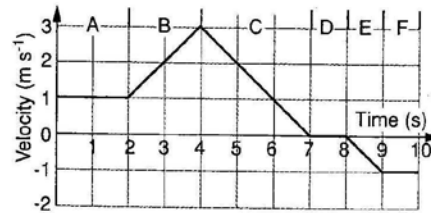
- 7 The following position–time graph is for a cyclist travelling along a straight road.



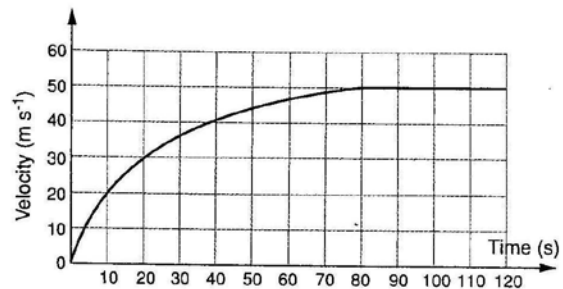
- Describe the motion of the cyclist.
  - What was the velocity of the cyclist during the first 30 s?
  - What was the cyclist's velocity during the final 10 s?
  - Calculate the cyclist's instantaneous velocity at 35 s.
  - What was the average velocity of the cyclist between 30 s and 40 s?
- 8 Which of the velocity–time graphs A–E best represents the motion of:
- a car coming to a stop at a traffic light?
  - a swimmer moving with constant speed?
  - a cyclist accelerating from rest with constant acceleration?
  - a car accelerating from rest and changing through its gears?



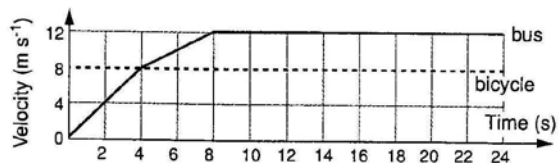
The following information relates to questions 9–11. The following graph shows the motion of a dog running along a footpath. In this problem, north is considered to be positive.



- Describe the motion of the dog during these sections of the graph.
  - A
  - B
  - C
  - D
  - E
  - F
- Calculate the displacement of the dog after:
  - 2 s
  - 7 s
  - 10 s.
- Plot a position–time graph of the dog's motion.
- The straight-line motion of a high speed intercity train is shown below.



- How long does it take the train to reach its cruising speed?
  - What is the acceleration of the train 10 s after starting?
  - What is the acceleration of the train 40 s after starting?
  - What is the displacement of the train after 120 s?
- 13 The velocity–time graphs for a bus and a bicycle travelling along the same straight stretch of road are shown below. The bus is initially at rest and starts moving as the bicycle passes it.



- Calculate the initial acceleration of the bus.
  - When does the bus first start gaining ground on the bicycle?
  - At what time does the bus overtake the bicycle?
  - How far has the bicycle travelled before the bus catches it?
  - What is the average velocity of the bus during the first 8 s?
- 14
- Draw an acceleration–time graph for the bus discussed in question 13.
  - Use your acceleration–time graph to determine the change in velocity of the bus over the first 8 s.

# Answers

1 a +4 m    b A, C    c B    d D

2 The car initially moves in a positive direction and travels 8 m in 2 s. It then stops for 2 s. The car then reverses direction for 5 s, passing back through its starting point after 8 s. It travels a further 2 m in a negative direction before stopping after 9 s.

3 a +8 m    b +8 m    c +4 m    d -2 m    4 8 s

5 a +4 m s<sup>-1</sup>    b 0    c -2 m s<sup>-1</sup>    d -2 m s<sup>-1</sup>    e -2 m s<sup>-1</sup>

6 a 18 m    b -2 m

7 a The cyclist travels with a constant velocity in a positive direction for the first 30 s, travelling 150 m during this time. Then the cyclist speeds up for 10 s, travelling a further 150 m. Finally the cyclist maintains this increased speed for the final 10 s, covering another 200 m in this time.

b +5 m s<sup>-1</sup>    c +20 m s<sup>-1</sup>    d approx. 13 m s<sup>-1</sup>    e +15 m s<sup>-1</sup>

8 a B    b A    c C    d D

9 a Running north at 1 m s<sup>-1</sup>

b Increasing speed from 1 m s<sup>-1</sup> to 3 m s<sup>-1</sup> while running north

c Running north but slowing to a stop

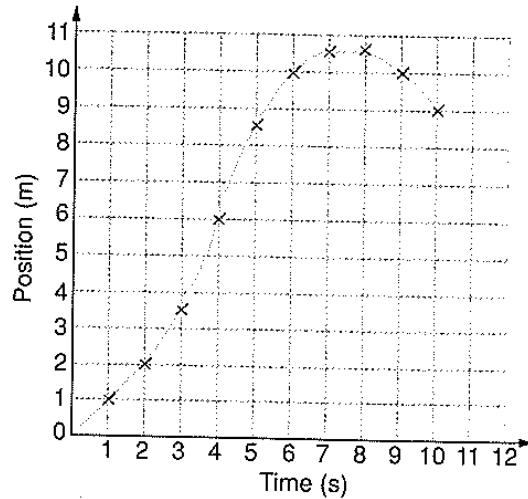
d Stationary

e Accelerating from rest to 1 m s<sup>-1</sup> while running south

f Running south at 1 m s<sup>-1</sup>

10 a 2 m north    b 10.5 m north    c 9 m north

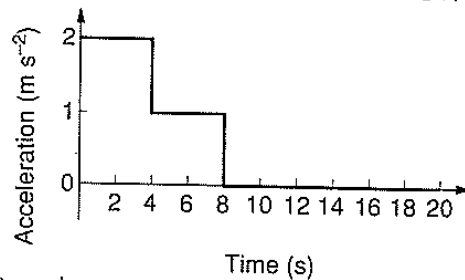
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12 a 80 s    b -1.3 m s<sup>-2</sup>    c -0.5 m s<sup>-2</sup>    d -4900 m

13 a +2 m s<sup>-2</sup>    b 4 s    c 10 s    d 80 m    e +7 m s<sup>-1</sup>

14 a



b +12 m s<sup>-1</sup>