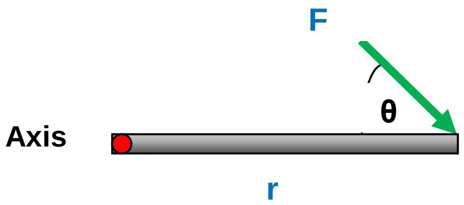
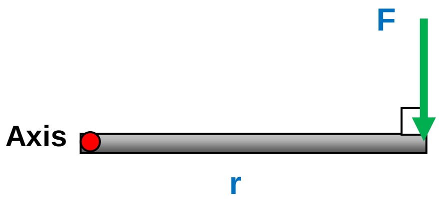
ENST1.1: MOMENT (OR TORQUE)

Moment

When a net force is applied through a point that is not the centre of mass, a moment is applied and rotation occurs. The turning force is most effective when it is acting at right angles to the rotating object, such as a beam (see below). For example, the turning force below right produces a greater moment than the situation below left because the force is at right angles to the beam.





The product of the force applied F and the distance from the axis of rotation, called the moment arm r, gives the magnitude of the moment M, or

$$M = F\sin\theta \times r$$

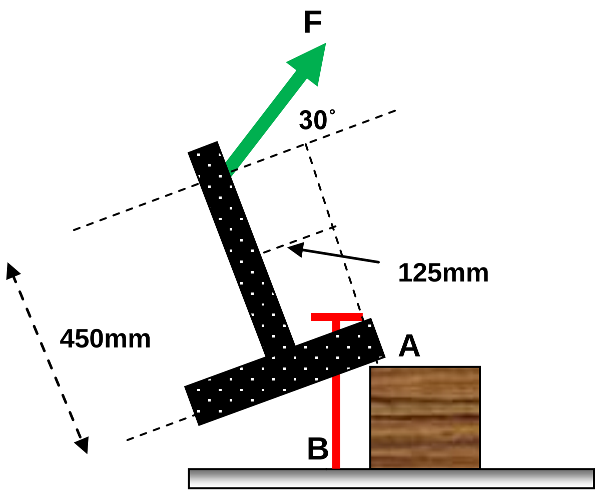
$$M = F_{\perp} \times r$$

where F sin θ is the perpendicular component of F to the moment arm.

Example

(Hibbeler, R.C., 2010, *Statics* 12th Ed. Pearson)

The force F exerted on the handle of the hammer below must produce a clockwise moment of 60Nm about point A. Determine the magnitude of F.



Solution

Take clockwise moments as negative

F has 2 components: F cos 30 and F sin 30

The sum of the moments of F cos 30 and F sin 30

about A are responsible for a clockwise moment of

60 NM, or $\Sigma M_A = \Sigma(F_{\perp} \times r)_A$ That is:

$-60\text{ Nm} = -F\cos 30 \times (0.45) - F\sin 30 \times (0.125)$

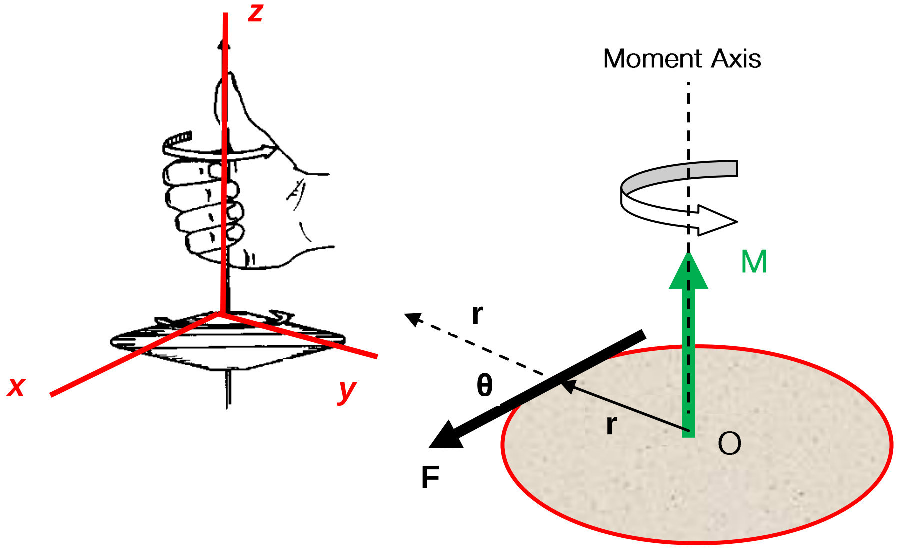
F = 132.7N

Moment as a vector cross product

The Moment of a force can also be written as a vector cross product

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Vector **M** has a direction that is perpendicular to the plane containing **r** and **F**. (see diagram below right)



The direction of the moment **M** is given by the right-hand rule (see diagram above left). Curling the right hand fingers from **r** toward **F** (**r** *cross* **F**) gives an upward-pointing thumb which is perpendicular to the plane containing **r** and **F**.

Note: **r** is a position vector relative to O. The order "**r** toward **F**" is important.

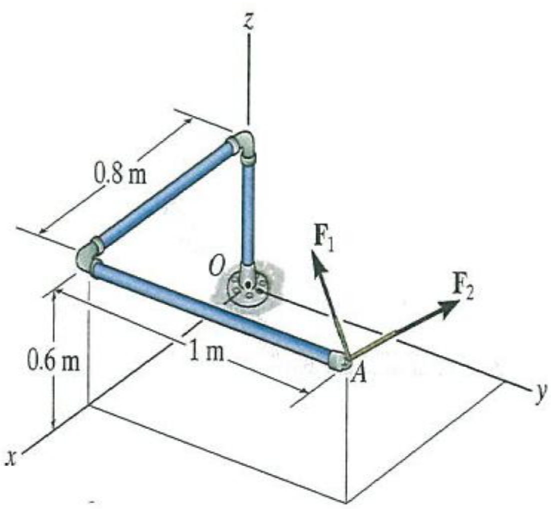
Moment as a Cartesian vector in 3 dimensions

If **r** and **F** are written as *x, y* and *z* coordinates (see diagram above left), then **M** can be found by evaluating the determinant

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Example (Hibbeler, R.C., 2010, *Statics* 12th Ed. Pearson)

If F_1 = {100**i** – 120**j** + 75**k**}N and F_2 = {–200**i** + 250**j** + 100**k**}N, determine the resultant moment produced by these forces about point O in the diagram below. Express as a Cartesian vector.



Solution

$F_R = F_1 + F_2$

F_R = {100**i** – 120**j** + 75**k**} + {–200**i** + 250**j** + 100**k**}

F_R = {– 100**i** + 130**j** +175**k**}N

r_A = {0.8**i** + 1**j** + 0.6**k**}m

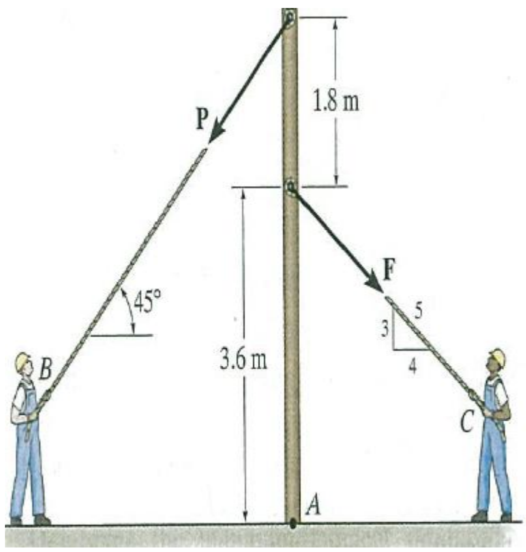
$$M = r_A \times F_R = \begin{vmatrix} i & j & k \\ 0.8 & 1 & 0.6 \\ -100 & 130 & 175 \end{vmatrix}$$

M = +{(1 ×175) – ((0.6 ×-130)}**i** – {(0.8×175) – ((0.6 ×-100)}**j** + {(0.8×130) – (1×-100)}**i**

M = {– 100**i** – 130**j** + 175**k**}Nm

Exercise (Hibbeler, R.C., 2010, *Statics* 12th Ed. Pearson)

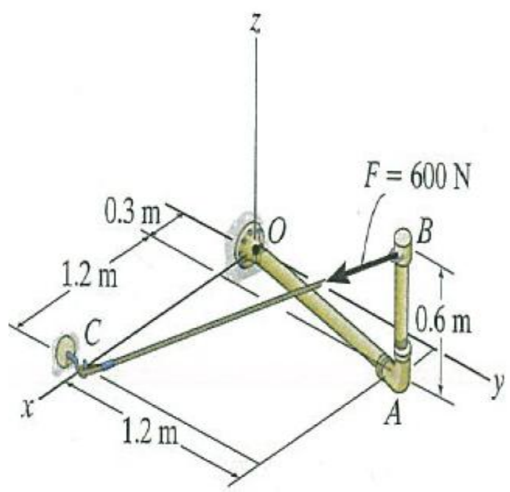
1. If the man at B exerts a force of P = 150N on his rope determine the magnitude of the force F the man at C must exert to prevent the pole from rotating , i.e so the resultant moment about A of both forces is zero. (Ans: 198.9N)



2. Determine the moment of force F about point O.

Express the result as a Cartesian vector.

(Ans: M = {300**j** – 600**k**}Nm)



Recall: $\vec{F}_{BC} = |\vec{F}_{BC}| \times \hat{r}_{BC}$

See also “Forces in 3 dimensions” tip on Learning Lab