

STUDY TIPS

Applications of Differentiation

DN1.9: MAXIMA AND MINIMA

As we have seen in curve sketching the maximum or minimum points of a function occur where the derivative is zero. We can therefore use calculus to solve problems that involve maximizing or minimizing functions.

Examples

1. The distance, s km, of a ship from a lighthouse at any time, t hours, is given by the formula $s = 8t - 2.5t^2$. When is the ship closest to the lighthouse and what is the minimum distance from the lighthouse?

$$s = 8t - 2.5t^2$$

$$\frac{ds}{dt} = 8 - 5t$$

The minimum distance will occur when $\frac{ds}{dt} = 0$:

$$\text{ie. } 8 - 5t = 0$$

$$5t = 8$$

$$t = 1.6$$

$$\begin{aligned} \text{When } t = 1.6: s &= 8(1.6) - 2.5(1.6)^2 \\ &= 6.4 \end{aligned}$$

\therefore The ship is closest to the lighthouse after 1.6 hours and the minimum distance is 6.4 km.

2. The sum of two numbers is 10. Find the numbers given that their product is a maximum.

Let the numbers be a and b and the product P .

$$\text{Then } a + b = 10 \Rightarrow b = 10 - a$$

$$\text{Also } P = a \times b$$

$$\text{ie } P = a(10 - a)$$

$$P = 10a - a^2$$

$$\text{and } \frac{dP}{da} = 10 - 2a$$

.....cont.

The maximum value of P will occur where $\frac{dP}{da} = 0$:

$$\text{ie } 10 - 2a = 0$$

$$10 = 2a$$

$$a = 5$$

$$\text{If } a = 5 \text{ then } b = 10 - a$$

$$= 10 - 5$$

$$= 5$$

\therefore The numbers are both 5

Exercises

1. Find two positive numbers whose sum is 18 such that the sum of their squares is a minimum.
2. Find the minimum value of the function $f(x) = 5x^2 - 30x + 17$
3. A ball is thrown vertically upward. The height, $h(t)$ m above the ground is a function of time with the formula
$$h(t) = 15t - 5t^2 + 1.6$$
Find the greatest height reached.
4. What is the maximum area that can be enclosed if a rectangle is created with a piece of wire 48 cm long?
5. The annual profit P made on a certain type of garment is related to the number n that are produced by the formula $P(n) = 300n - 7200 - 0.2n^2$.
How many garments should be produced to maximize profit?
6. A small open box is to be made by cutting squares from the corners of a square sheet of cardboard and folding up the sides. If the cardboard measures 10cm x 10cm what side length should the cut squares have so that the volume of the box is a maximum? [Hint: draw a diagram].

Answers

1. The two numbers are both 9.
2. -28
3. 12.85m
4. 144cm²
5. 750
6. 5/3 cm