MA1.1: INTRODUCTION TO MATRICES

A matrix is a **rectangular array** of **elements**.
Matrices are usually denoted by upper case letters.
The **elements** are usually written within brackets.
The **order** or shape of the matrix is determined by the number of rows and columns of the matrix. The number of rows is always given first.

Example. $A = \begin{pmatrix} 1 & 2 & -9 \\ 2 & 5 & -3 \end{pmatrix}$

A has two rows and 3 columns and is called a 2 x 3 matrix
A matrix with *m* **rows** and *n* **columns** is called a matrix of **order** *m* x *n.*

Square matrix

A matrix with the same in which the number of rows equals the number of columns is called a square matrix.

Example $B = \begin{pmatrix} 2 & 3 \\ 2 & 5 \end{pmatrix}$

B is a square 2 x 2 matrix

Unit Matrix

An unit (or identity) matrix is a square matrix with diagonal elements equal to one, and all other elements equal to zero. The unit matrix is usually denoted by I.
I_3 is a 3 x 3 unit matrix.

Example $I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Row Matrix

A matrix with **one row** is called a **row matrix**.

Example D = (2, 1, 0, 4) is a 1 x 4 row matrix

Column Matrix

A matrix with **one column** is called a **column matrix**.

Example $E = \begin{pmatrix} 2 \\ -4 \\ 1 \end{pmatrix}$ is a 3 x 1 column matrix

Zero Matrix

A zero matrix has all elements equal to zero. A zero matrix can be written as 0.

Example $0_2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is a 2 x 2 zero matrix

See exercise 1

Equal Matrices

For two matrices to be equal they must be the same shape and the corresponding elements must be equal.

Examples

1. If A equals B and $A = \begin{pmatrix} 1 & 3 & 5 \\ 3 & 7 & 2 \\ 8 & 0 & -2 \end{pmatrix}$ then $B = \begin{pmatrix} 1 & 3 & 5 \\ 3 & 7 & 2 \\ 8 & 0 & -2 \end{pmatrix}$

2. C = (4, 3,) and D = $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ are not equal

   C is not equal to D because they do not have the same shape even though the numbers are the same.

3. Given: $A = \begin{pmatrix} 2 & 5 & b \\ 5 & 3 & 1 \\ 2 & 0 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 5 & 7 \\ 5 & a & 1 \\ 2 & 0 & -2 \end{pmatrix}$

   If A and B are equal then a = 3 and b = 7

See exercise 2

Exercises

Exercise 1

1.Write down the order of the following matrices.

(a) $\begin{pmatrix} 7 & -5 & 0 \\ 6 & 2 & -1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 2 \\ 1 & 1 \end{pmatrix}$ (c) $\begin{pmatrix} 2 \\ -4 \\ 1 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 1 \\ 3 & 0 \\ -2 & 3 \end{pmatrix}$

2. (a) Write down a 2 x 2 identity matrix.

   (b) Write down a 3 x 3 zero matrix.

Exercise 2

(a).Which of the following matrices are equal?

$$A = \begin{pmatrix} 3 & 0 \\ 1 & -2 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} 3 & 0 \end{pmatrix} \qquad D = \begin{pmatrix} 3 & 0 \\ 1 & -2 \end{pmatrix}$$

$$E = \begin{pmatrix} 3 & 5 & 1 \\ 2 & 0 & 1 \end{pmatrix} \qquad F = \begin{pmatrix} 0 & 3 \end{pmatrix} \qquad G = \begin{pmatrix} 3 & 5 & 1 \\ 2 & 0 & 1 \\ 1 & 3 & 0 \end{pmatrix}$$

(b) Find *a* and *b* given that H = E (above)

$$H = \begin{pmatrix} 3 & 5 & b \\ 2 & a & 1 \end{pmatrix}$$

Answers

Exercise 1

1(a) 2 x 3, (b) 2 x 2 (c) 4 x 1 (d) 3 x 2

2(a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Exercise 2

(a) A and D (b) a = 0, b = 1