

## STUDY TIPS

# ET1.1: LINEAR EQUATIONS

Equations with one variable may be solved using transposition skills to make the variable the subject of the equation.

Using a sequence of inverse operations to 'undo' the equation

$$3(2x + 7) = -15 \dots$$

.... becomes....

$$x = -6$$

## Simple equations

### Examples

1.  $m + 4 = -2$   
 $m + 4 - 4 = -2 - 4$  [subtract 4 both sides]  
 $m = -6$

2.  $p - 2 = 5$   
 $p - 2 + 2 = 5 + 2$  [add 2 both sides]  
 $p = 7$

3.  $3g = 18$   
 $\frac{3g}{3} = \frac{18}{3}$  [divide 3 both sides]  
 $g = 6$

4.  $\frac{y}{4} = -5$   
 $\frac{y}{4} \times 4 = -5 \times 4$  [multiply by 4 both sides]  
 $y = -20$

## More complex equations

With more complicated equations more than one inverse operation may need to be applied

### Examples

$$\begin{aligned} 1. \quad 2w - 3 &= -17 \\ 2w &= -14 && \text{[add 3 to both sides]} \\ w &= -7 && \text{[divide 2 both sides]} \end{aligned}$$

$$\begin{aligned} 2. \quad \frac{3d}{4} + 5 &= 7 \\ \frac{3d}{4} &= 2 && \text{[subtract 5 each side]} \\ 3d &= 8 && \text{[multiply 4 each side]} \\ d &= \frac{8}{3} && \text{[divide 3 each side]} \end{aligned}$$

With practice more than one operation can be done in the same line:

$$\begin{aligned} 3. \quad 3c + 1 &= c - 5 && \text{[move all terms with 'c' to one side of the equation]} \\ 3c - c &= -5 - 1 && \text{[subtract c, subtract 1 each side]} \\ 2c &= -6 \\ c &= -3 && \text{[divide 2 each side]} \end{aligned}$$

Look at the examples below for techniques to deal with equations that contain brackets and fractions

$$\begin{aligned} 4. \quad 3(5 - 2j) &= 33 && \text{[expand the brackets first]} \\ 15 - 6j &= 33 \\ -6j &= 18 && \text{[subtract 15 each side]} \\ j &= -3 && \text{[divide (-6) each side]} \end{aligned}$$

$$\begin{aligned} 5. \quad 2(3k - 1) &= 5(k + 7) \\ 6k - 2 &= 5k + 35 \\ 6k - 5k &= 35 + 2 && \text{[subtract 5k, add 2 each side]} \\ k &= 37 \end{aligned}$$

With just one fraction on each side of the equality use *cross multiplication*:

$$\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$$

$$\begin{aligned} 6. \quad \frac{h+1}{3} &= \frac{h}{4} \\ 4(h+1) &= 3h && \text{[cross multiplying]} \\ 4h + 4 &= 3h && \text{[expand brackets]} \end{aligned}$$

$$h = -4 \quad \text{[subtract } 3h, \text{ subtract } 4 \text{ each side]}$$

$$7. \quad \frac{2z+11}{7} = \frac{z-3}{12}$$

$$12(2z+11) = 7(z-3) \quad \text{[cross multiplying]}$$

$$24z + 132 = 7z - 21 \quad \text{[expand brackets]}$$

$$17z = -153 \quad \text{[subtract } 7z, \text{ subtract } 132 \text{ each side]}$$

$$z = -9$$

When cross multiplication is not appropriate fractions may be removed by multiplying both sides of the equation by the *lowest common multiple (LCM)* of all the denominators:

$$8. \quad \frac{3u}{4} - \frac{1}{3} = 7 \quad \text{[NB: } 7 = \frac{7}{1} \therefore \text{ LCM for } 4, 3 \text{ and } 1 \text{ is } 12]$$

$$\frac{3u}{4} \times 12 - \frac{1}{3} \times 12 = 7 \times 12 \quad \text{[multiply each side by the LCM]}$$

$$9u - 4 = 84 \quad \text{[simplify]}$$

$$u = 88$$

$$u = \frac{88}{9}$$

## Exercise

1 Solve the following equations

$$\text{a) } x + 3 = 7 \quad \text{b) } 5 - j = -2 \quad \text{c) } 3c = 12$$

$$\text{d) } -r = -12 \quad \text{e) } \frac{m}{2} = -7 \quad \text{f) } -8u = 12$$

$$\text{g) } 4g + 4 = 16 \quad \text{h) } 7 - 2w = 1 \quad \text{i) } \frac{e}{2} - 5 = -8$$

$$\text{j) } 21 - 3t = 12 \quad \text{k) } \frac{y}{5} - 9 = -5 \quad \text{l) } 3 - \frac{u}{2} = -7$$

2 Solve these equations

$$\text{a) } 5i + 2 = i + 10 \quad \text{b) } 10p - 11 = 2p - 3$$

$$\text{c) } 5a - 12 = 3a + 6 \quad \text{d) } 10d + 10 = 3d + 3$$

$$\text{e) } f + 6 = 6f - 9 \quad \text{f) } 8 - g = 5g + 14$$

$$\text{g) } 5h - 2 = 7h - 12 \quad \text{h) } 6j + 13 = 4j + 13$$

3 Solve

$$\text{a) } 3(2k - 4) = 18 \quad \text{b) } 5(2z + 9) = 15$$

$$\text{c) } 3(x + 4) = 6 \quad \text{d) } 3(c + 3) + 2(c - 5) = 4$$

$$\text{e) } 3(2v - 3) + 2(v - 4) = -25 \quad \text{f) } 3(b + 4) = 2(4b + 1)$$

4 Solve

a)  $\frac{9n}{5} - 4 = 5$

c)  $1 - \frac{9q}{2} = -8$

e)  $\frac{3-2e}{11} = 1$

g)  $\frac{3t}{8} + 4 = 1$

i)  $\frac{5u-4}{4} = \frac{u-5}{5}$

k)  $\frac{p+1}{3} + 1 = 4$

m)  $\frac{d-3}{3} - 4 = \frac{d-2}{2}$

b)  $\frac{4m}{3} - 11 = 9$

d)  $\frac{w-4}{2} = 2$

f)  $\frac{3r+9}{5} = -3$

h)  $\frac{y}{3} = \frac{2}{9}$

j)  $\frac{2i+1}{7} = \frac{3i-5}{4}$

l)  $2 - \frac{5a-4}{6} = 4$

n)  $\frac{1-m}{5} - m = \frac{2m-1}{2}$

## Answers

- |   |       |       |       |                   |        |                   |       |                  |
|---|-------|-------|-------|-------------------|--------|-------------------|-------|------------------|
| 1 | a) 4  | b) 7  | c) 4  | d) 12             | e) -14 | f) $-\frac{3}{2}$ | g) 3  | h) 3             |
|   | i) -6 | j) 3  | k) 20 | l) 20             |        |                   |       |                  |
| 2 | a) 2  | b) 1  | c) 9  | d) -1             | e) 3   | f) -1             | g) 5  | h) 0             |
| 3 | a) 5  | b) -3 | c) -2 | d) 1              | e) -1  | f) 2              |       |                  |
| 4 | a) 5  | b) 15 | c) 2  | d) 8              | e) -4  | f) -8             | g) -8 | h) $\frac{2}{3}$ |
|   | i) 0  | j) 3  | k) 8  | l) $-\frac{8}{5}$ | m) -24 | n) $\frac{7}{22}$ |       |                  |