# DN1.2: LIMITS

The ***limit*** of a function describes the *behaviour* of a function as the variable approaches a particular value.

## Examples

1. *Find the limit of the function f(x) = x + 2 as x approaches 2*

The behaviour of f(x) as x $\rightarrow$ 2 is shown in the table:

| x | 1.95 | 1.995 | $\rightarrow 2 \leftarrow$ | 2.005 | 2.05 |
|---|---|---|---|---|---|
| f(x) | 3.95 | 3.995 | $\rightarrow 4 \leftarrow$ | 4.005 | 4.05 |

The table shows that as x approaches 2, f(x) approaches 4

$\therefore \quad \lim_{x \to 2} x + 2 = 4$

2. *Find* $\lim_{h \to 0} (5x^2 + 2xh + h)$

The 2xh and h terms will approach zero as h approaches zero.
The limit can be found by substituting zero for h:.

$$\lim_{h \to 0} (5x^2 + 2xh + h) = 5x^2 + 2x.0 + 0$$
$$= 5x^2$$

3. *Find* $\lim_{h \to 0} \frac{\sin(h)}{h}$

It is not possible to find the limit by substituting h = 0

But consider the behaviour of f(h) = $\frac{\sin(h)}{h}$ as h $\rightarrow$ 0:

| h | -0.5 | -0.3 | -0.2 | -0.1 | $\rightarrow 0 \leftarrow$ | 0.01 | 0.1 | 0.2 | 0.3 | 0.5 |
|---|---|---|---|---|---|---|---|---|---|---|
| f(h) | 0.959 | 0.985 | 0.993 | 0.998 | $\rightarrow ? \leftarrow$ | 0.99998 | 0.998 | 0.993 | 0.985 | 0.959 |

It appears that $\lim_{h \to 0} \frac{\sin(h)}{h} = 1$

## Exercises

Determine the limit of the following:

1) $\lim_{h\to 0}(4xh+3)$
2) $\lim_{x\to 0}\frac{9-x^2}{4}$
3) $\lim_{h\to 0}\frac{xh-2h}{h}$
4) $\lim_{x\to 0}\frac{\tan(x)}{x}$

## Answers

1) 3
2) $\frac{9}{4}$
3) x – 2
4) 1