LT1.1: LAPLACE TRANSFORMS

BASIC DEFINITION

In your study of Differential Equations so far you have probably solved first and second order equations using methods such as separation of variables, substitutions, homogeneous equations, and integrating factor technique.

Transforms are another means of solving some differential equations that may prove too difficult to solve using the above mentioned methods.

Assumed knowledge for studying Laplace transforms: Differentiation
Integration
Partial fractions
Algebra & Transposition
Exponentials & Logarithm's

The fundamental rule for Laplace Transforms is:

$$\boxed{L[y(t)] \;=\; Y(s) \;=\; \int_0^\infty e^{-st} y(t)dt}$$

e.g. 1

Determine the Laplace Transform of $y(t) = \text{ constant } C$ using the basic definition.

Sol$^{\underline{n}}$:

$$\therefore \qquad L[y(t)] = \int_0^\infty e^{-st}.C\,dt$$

$$= C\int_0^\infty e^{-st}dt$$

$$= C\left[-\frac{1}{s}.\quad e^{-st}\right]_0^\infty$$

$$= \left(-\frac{C}{s}.e^{-\infty}\right) - \left(-\frac{C}{s}.e^{0}\right)$$

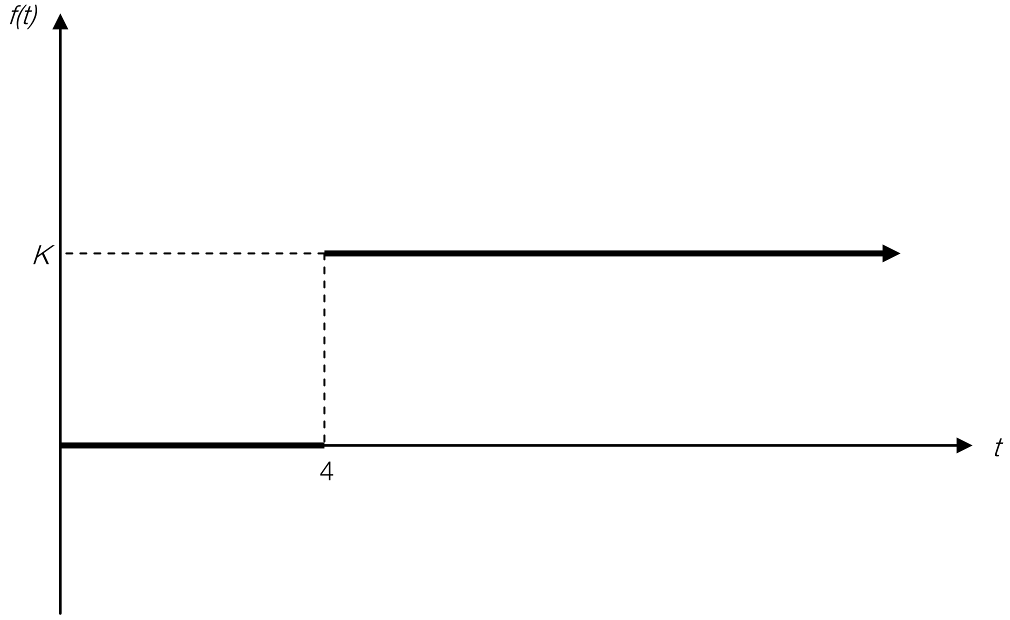
$$e^{-\infty} = 0 \;\;\&\;\; e^{0} = 1$$

$$= (0) - \left(-\frac{C}{s}\times 1\right)$$

$$= \frac{C}{s}$$

e.g. 2

A signal that is graphed is given below:



Determine the Laplace transform of the above signal.

Sol$^{\underline{n}}$:

From the graph we define $f(t)$ as:

$$f(t)=\begin{cases}0 & 0\le t<4\\ K & t>4\end{cases}$$

Therefore the transform can be separated into 2 parts.

$$\begin{aligned}
\int_0^\infty e^{-st}f(t)dt &= \int_0^4 e^{-st}f(t)dt+\int_4^\infty e^{-st}f(t)dt\\
&=\int_0^4 e^{-st}(0)dt+\int_4^\infty e^{-st}(K)dt\\
&=0+K\int_4^\infty e^{-st}dt\\
&=K\left[-\frac{1}{s}.e^{-st}\right]_4^\infty\\
&=\left(-\frac{K}{s}.e^{-\infty}\right)-\left(-\frac{K}{s}.e^{-4s}\right)\\
&=(0)-\left(-\frac{K}{s}.e^{-4s}\right)
\end{aligned}$$

$$\therefore \qquad L[f(t)]=\frac{K}{s}.e^{-4s}$$