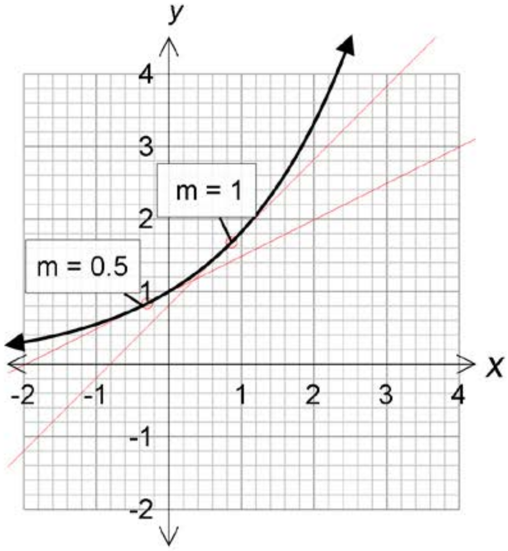
DN1.3: GRADIENTS, TANGENTS AND DERIVATIVES

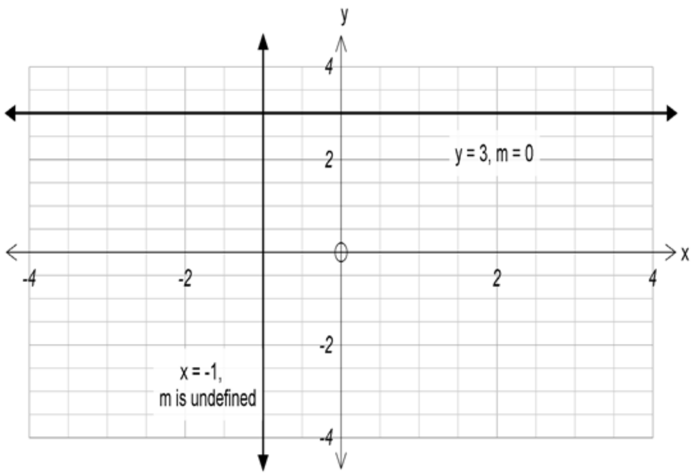
Gradient of a Curve

The gradient at a point on a *curve* is the gradient of the *tangent* to the curve at that point.



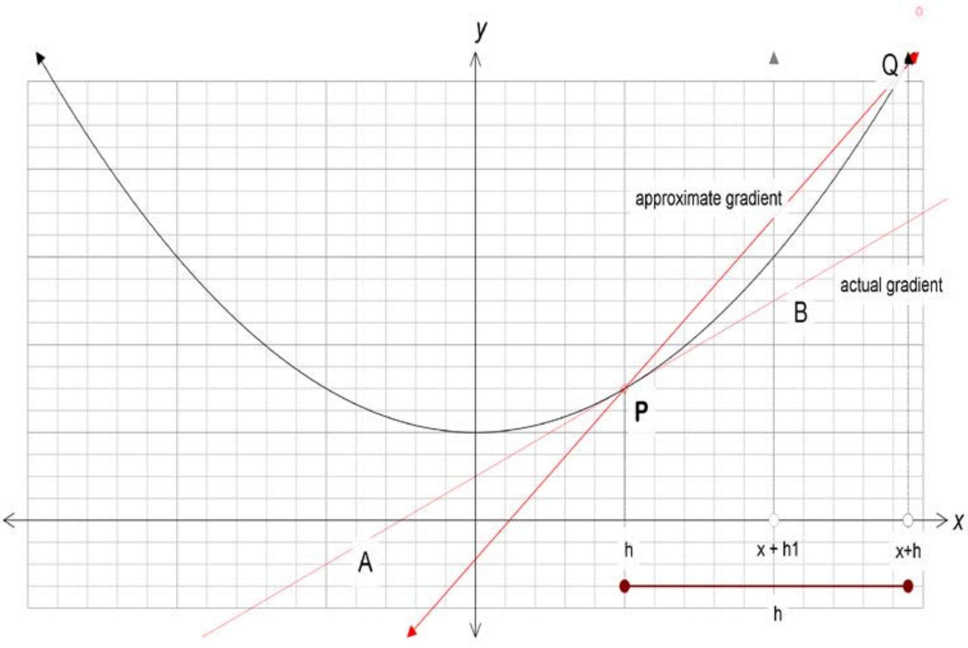
For the curve y = f(x) below, the gradient at point P is the gradient of the line AB.

Special cases: horizontal and vertical lines



A line parallel to the x-axis with equation of the form y = k (k constant), has a gradient of zero.

As a line becomes closer to vertical it's gradient gets larger and larger. A line parallel to the y-axis with equation of the form x = c (x constant) has a gradient which is undefined.



This gradient cannot be calculated - only one point on the line is known. The gradient of the line PQ **can** be calculated and this can be used to approximate the gradient of AB.

$$\text{The gradient of } PQ = \frac{f(x+h) - f(x)}{h}$$

As the value of *h* decreases (i.e Q becomes closer to the point P), the approximation of the gradient is more accurate. The value of the gradient becomes most accurate as *h* approaches zero.

> The gradient formula for the curve y = f(x) is defined as the derivative function
>
> $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}, \quad h \neq 0$$

The derivative function $f'(x)$ gives the slope of the tangent to the curve $f(x)$ at any point *x.*

Example

1. If the derivative function of $f(x) = \frac{3}{x}$ is $f^{1}(x) = \frac{-3}{x^2}$, find the slope of the tangent to the curve

 at x = 4

$$\text{At } x = 4, \quad f^{1}(x) = f'(4)$$

$$= \frac{-3}{4^2}$$

$$= \frac{-3}{16}$$

Exercises

1. If the derivative function for $f(x) = x^3 - x$ is $f'(x) = 3x^2 - 1$, find the slope of the tangent to this curve at
   a) x = 2
   b) x = 0
   c) x = -9

2. If the derivative function of $f(x) = \sin(x)$ is $f'(x) = \cos(x)$ find the gradient of y = sin(x) at
   a) x = 0
   b) $x = \frac{\pi}{2}$
   c) x = 3.5

3. Determine $\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$ and hence find the slope of the tangent to the curve $y = x^2$ at
   a) x = 2
   b) x = 0
   c) x = -9

Answers

1. a) 11
   b) -1
   c) 242

2. a) 1
   b) 0
   c) -0.94

3. $\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = 2x$
   a) 4
   b) 0
   c) -18