RMIT UNIVERSITY

PF1.3 FORCES: STATICS

Static Equilibrium

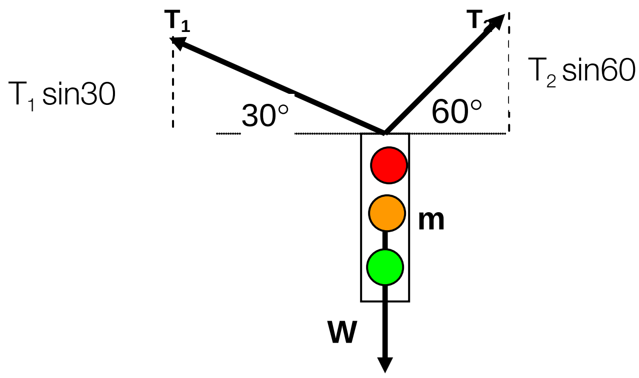
Structures that are stable are said to be in static equilibrium. There are two types of static equilibrium: translational equilibrium and rotational equilibrium.

Translational equilibrium means that the sum of the forces (or net force) acting on the system is zero, ie, $\Sigma F = 0$, or:

Forces up	=	Forces down
Forces left	=	Forces right

Example 1

Consider a set of traffic lights of mass m suspended above an intersection by a pair of wires of negligible mass, as shown below.



Since this structure is in equilibrium the magnitude of the components of forces upwards must equal the magnitude of the components of forces downwards, or

Forces up = Forces down (there is no vertical motion)

Resolving T_1 and T_2 vertically in the diagram above:

$T_1 \sin 30 + T_2 \sin 60 = W = mg$

Alternatively, we could resolve T_1 and T_2 horizontally.
Since there is no horizontal motion:

Forces left = Forces right

Resolving T_1 and T_2 horizontally in the diagram above:

$T_1 \cos 30 = T_2 \cos 60$

Note that the vertical weight force W (= mg) when resolved horizontally equals zero ($W \times \cos 90 = 0$).

Rotational equilibrium

Torque or Moment of a Force

The turning effect or torque τ of a force F acting at a distance r from the axis of rotation of a rigid body is given by

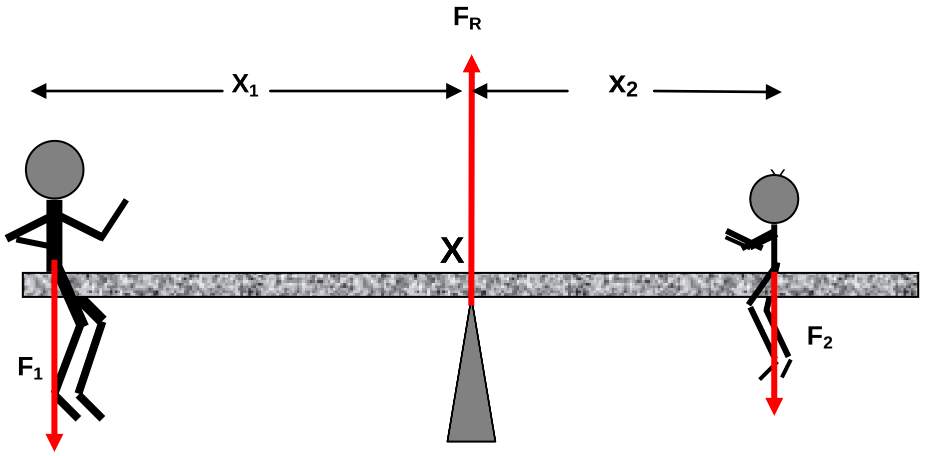
$$\tau = F \times r$$

where F and r are mutually perpendicular (at right angles) to each other.

For rotational equilibrium to occur the sum of the torques (or net torque) acting on the structure is zero, ie $\Sigma\tau = 0$, or

$$\text{Clockwise Torques} = \text{Counterclockwise Torques}$$

Example 2 Consider the seesaw example below.



For translational equilibrium: $\Sigma F = 0$ or Forces up = Forces down

$$F_R = F_1 + F_2$$

For rotational equilibrium: $\Sigma\tau = 0$ or

Clockwise torques = Counterclockwise torques

Taking torques about X in the diagram above: $F_2 \times x_2 = F_1 \times x_1$

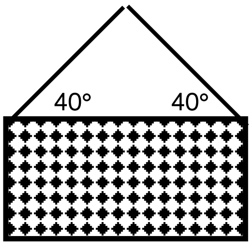
An example of a system that is not in rotational equilibrium is a **moving** see-saw. If you are sitting on one end and an elephant (!) decides to sit on the other end, then it will rotate!!

Exercise

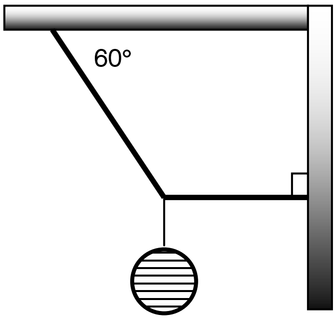
Take $g = 9.8ms^{-2}$

1. In Example 1, T_1 = 1000N, and T_2 = 2000N. Using vertical forces only calculate:
   (a) the maximum **weight** of the traffic lights that can be supported by the cables.
   (b) the maximum **mass** of the traffic lights that can be supported by the cables.

2. A picture is hung as shown in the following diagram. If the hanging wire has a breaking strength of 40 N, what is the maximum possible mass of the picture?



3. A 100 gram electric light is supported by two cables, one at an angle of 60° with the ceiling, and the other being perpendicular to the wall. Assuming the mass of each cable is negligible, calculate the tension in each cable.



4. Two children are balanced on a see-saw which is supported in the middle. One child weighs 200 N and is 1.2 m from the axis, while the other child is seated 1.5 m from the axis. How much does the second child weigh? Ignore the mass of the see-saw.

5. Two children wish to make a see-saw from a 5.0 m plank of wood. The children weigh 25 kg and 20 kg. They each wish to sit right on the ends of the plank. Where should the plank be supported in order for it to balance?

Answers

1. (a) $T_1 = 2.2\times10^3$N, (b) 224kg 2. T = 5.2kg 3. T1 = 1.13 N, T2 = 0.57 N.
4. W = 160 N 5. 2.2 m from the heavier child.