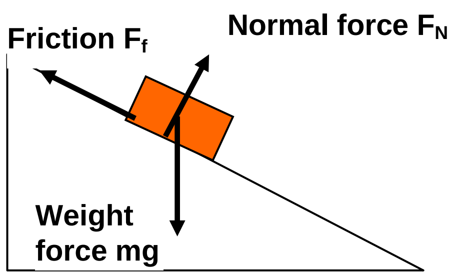
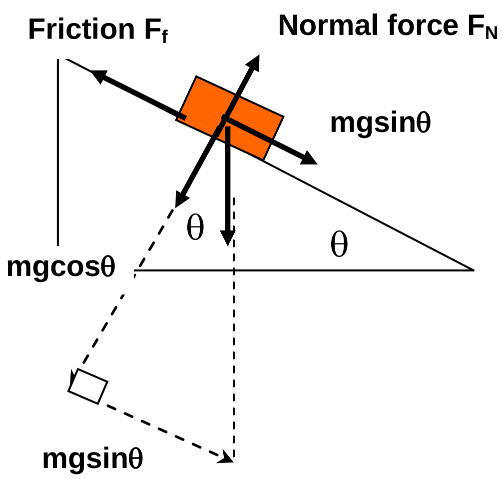
PF1.2: FORCES ON SLOPES

The normal force of an object placed on a sloping surface is always perpendicular to the surface. Another vital feature is that just as we can analyse the horizontal and vertical components of the motion of an object separately, we can look at components parallel to and perpendicular to the surface of a sloping surface as well. See diagrams below.

Forces acting on a block on an inclined plane



Weight force resolved into its components

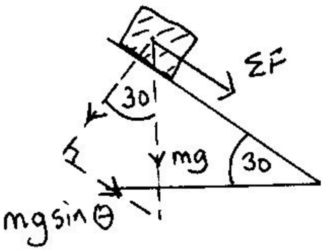


Note that with the above diagrams:

- Weight = mg; acts through the centre of mass.
- Normal force F_N is always at right angles to the surface.
- Friction acts to oppose sliding motion (eg, if the mass were being dragged uphill, friction would act downhill)
- The weight force is resolved into 2 components:
  - o mg cosθ perpendicular to plane, and
  - o mg sinθ parallel to the plane.
- The resultant force ΣF down the slope is given by $\Sigma F = mg\sin\theta - F_f$ where F_f is friction
- The resultant force ΣF perpendicular to the slope is zero, hence: $mg\cos\theta = F_N$

Example

A toy car of mass 50g travels down a smooth incline at 30 degrees to the horizontal. Friction may be ignored. Calculate (a) the net force acting on the car as it rolls down the slope, and (b) the force of the incline on the car as it travels down the slope.

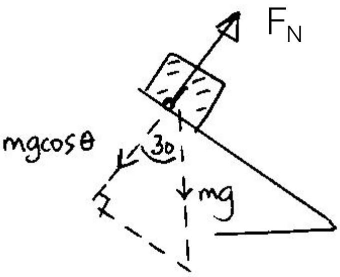


(a) $\Sigma F = ma = mg \sin\theta$
where $mg \sin\theta$ is the component of the force **parallel** to the slope.

Note: the surface is frictionless (smooth), therefore the only force allowing the car to roll down the incline is the component of the gravitational force '$mg \sin\theta$'.

$\Sigma F = m \times g \times \sin\theta = 50 \times 10^{-3} \times 9.8 \times \sin 30 =$ 0.25N

Note: grams have been converted into kilograms



(b) The force of the incline on the car is a force that acts **perpendicular** to the slope, ie, the normal force F_N

This is equal to $mg \cos\theta$

$F_N = mg \cos\theta = 50 \times 10^{-3} \times 9.8 \times \cos 30 = 0.43$N

Trivia

The steepest road in the world is in Dunedin, New Zealand. It has an incline of 52°. Ignoring friction, a car left with its handbrake off would accelerate down this street at
$a = g \times \sin\theta = 9.8 \times \sin 52^{\circ} = 7.7\text{ms}^{-2}$.

Exercise

Take $g = 9.8\text{ms}^{-2}$

1 A boy and his skateboard of total mass 60kg coasts down a **frictionless** ramp at an angle of 30° to the horizontal.
(a) Calculate the normal force acting on the boy and skateboard.
(b) Calculate the force acting on the boy and skateboard parallel to the ramp.

2 The boy now coasts down another ramp, but this time the ramp has a **rough** surface.
(a) Calculate the normal force acting on the boy and skateboard.
(b) Calculate the force acting on the boy and skateboard parallel to the ramp.
(c) If the ramp has a frictional force of 54N, what is the net force acting on the boy and the skateboard?
(d) Calculate the acceleration of the boy.
(e) If the boy started from rest, and the ramp is 4m long, what was the speed of the boy at the bottom of the ramp?

Answers

1. (a) 509N (b) 294N 2. (a) 509N (b) 294N (c) 240N (d) 4.0ms^{-2} (e) 5.6 ms^{-2}