Applications of Differentiation

DN1.8: CURVE SKETCHING

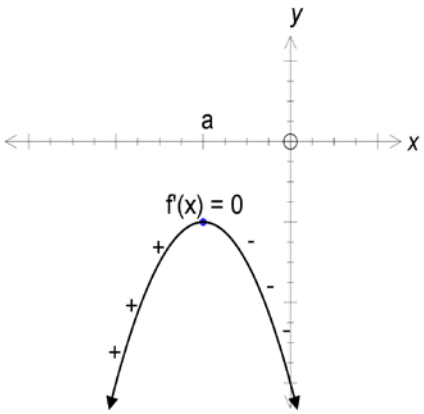
To sketch a curve, find

- the maximum and minimum stationary points
- the intercepts on the axes

A **stationary point** is a point on a graph of a function y = f(x) where the tangent to the curve is horizontal.
At a stationary point the derivative function **y = f '(x) = 0**.

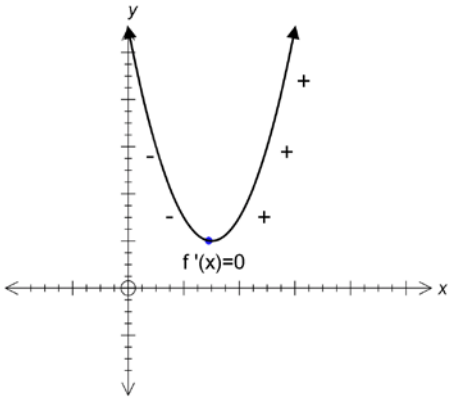
A **maximum** stationary point occurs at x = a if

- f '(x) < 0 for x < a
- f '(x) = 0 for x = a
- f '(x) > 0 for x > a



A **minimum** stationary point occurs at x = a if

- f '(x) < 0 for x < a
- f '(x) = 0 for x = a
- f '(x) > 0 for x > a



Example

Find the turning point of the parabola defined by $y = x^2 + 4x + 5$

$f(x) = x^2 + 4x + 5 \quad \Rightarrow \quad f'(x) = 2x + 4$

At a stationary point $f'(x) = 0$

ie $2x + 4 = 0$

$2x = -4$

$x = -2$

When $x = -2$, $y = (-2)^2 + 4(-2) + 5 = 1$

So there is a stationary point at (-2, 1).

Sign Test

Do a sign test to check whether the stationary point is a minimum or maximum. (Check the slope of the tangent on each side of the stationary point)

x	-2.1	-2	-1.9
f '(x)	-	0	+
gradient	\	—	/

∴ There is a minimum point at (-2,1)

Example

Sketch the graph of $y = x^3 - x$

$f(x) = x^3 - x \quad \Rightarrow \quad f'(x) = 3x^2 - 1$

Stationary points: $f'(x) = 0$

$3x^2 - 1 = 0$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm \frac{1}{\sqrt{3}}$

$x \approx \pm 0.58$

When $x = 0.58$, $y = -0.38$ (0.58,-0.38)

$x = -0.58$, $y = 0.38$ (-0.58, 0.38)

Do sign tests to check whether stationary points are *minima* or *maxima*:

x	0.5	0.58	0.6
f '(x)	-	0	+
gradient	\	—	/

x	-0.6	-0.58	-0.5
f '(x)	+	0	-
gradient	/	—	\

There is a minimum point at (0.58, -38) and a maximum point at (-0.58, 0.38)

x-intercepts: When $y = 0$, $x^3 - x = 0$

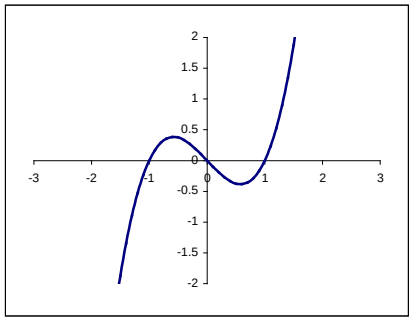
$x(x^2 - 1) = 0$

$x(x - 1)(x + 1) = 0$

x-intercepts at $x = 0$, $x = 1$ and $x = -1$

y-intercepts: When $x = 0$, $y = 0$

$y = x^3 - x$



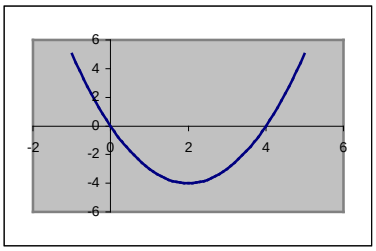
Exercise

Sketch the graphs of the following functions showing all intercepts and turning points

1. $y = x^2 - 4x$
2. $y = x^3 - 2x^2 + x$
3. $y = 6 - x - x^2$
4. $y = (x + 1)^4$

Answers

1.



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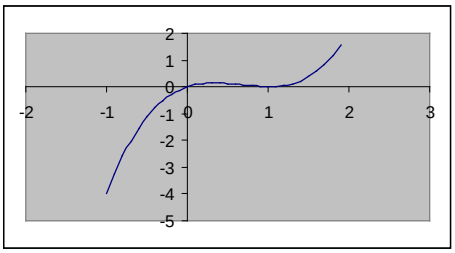

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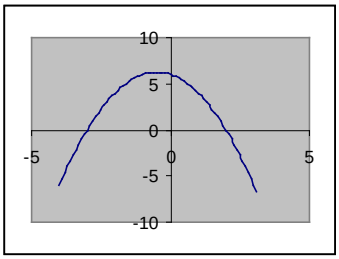

4.