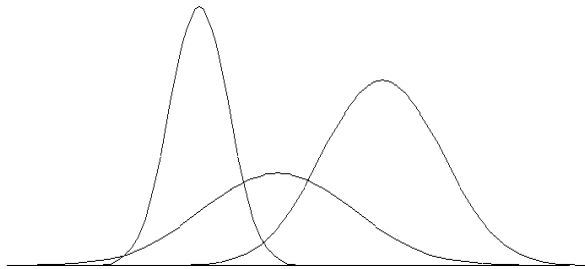
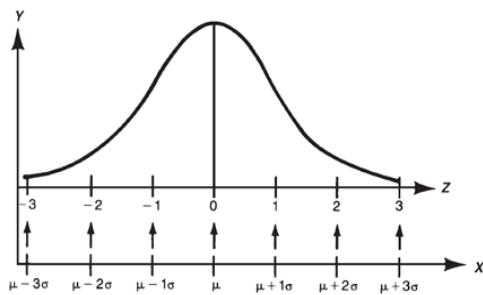


PROBABILITY AND THE NORMAL DISTRIBUTION

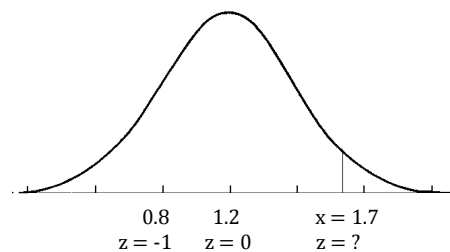
Even when data follows a normal distribution different data sets will have their own mean and standard deviation and a different bell shaped curve.



But every score in a normally distributed data set regardless of the shape has an equivalent score in the standard distribution. The mean of a normal distribution corresponds to a standardised score of 0 and we can see that $\mu \pm \sigma \rightarrow \pm 1$, $\mu \pm 2\sigma \rightarrow \pm 2$ and $\mu \pm 3\sigma \rightarrow \pm 3$.



BUT

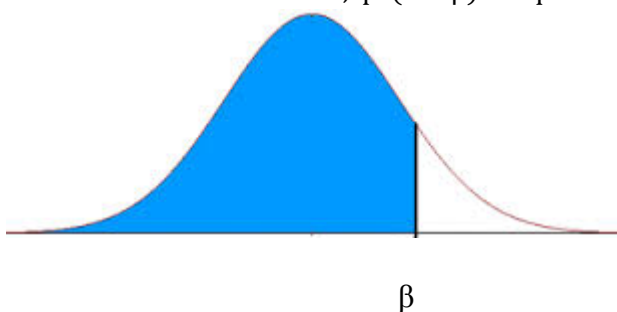


For other values we can use the formula
$$z = \frac{x - \mu}{\sigma}$$
 to find z scores.

To find the standardised score for $x = 1.7$:
$$z = \frac{x - \mu}{\sigma} = \frac{1.7 - 1.2}{0.4} = 1.25$$

A score of 1.7 in the distribution with mean 1.2 and standard deviation 0.4 is equivalent to a standardised score of 1.25. Alternatively, the score 1.7 is 1.25 standard deviations above the mean for that distribution.

Once we have converted the scores of our distribution into standard scores or z-scores we can use normal distribution tables to calculate precise percentages and probabilities. The normal distribution is a continuous distribution, so we can find the probability that x is greater than or less than a particular value, but not that x is equal to a particular value. Because the total area under the standardised curve is 1, $pr(z < \beta)$ is equivalent to the area to the left of β .



Examples

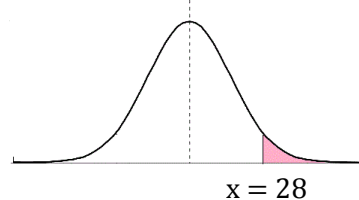
1. If the mean maximum temperature for Melbourne in January is 25.9°C with a standard deviation of 2.1 what is the probability that the mean maximum temperature for January 2015 will be above 28°C ?

First draw a diagram:

Then standardise $x = 28$

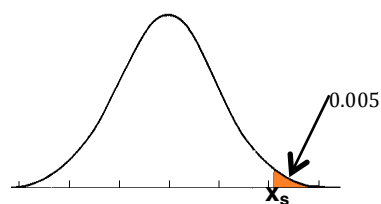
$$z = \frac{x - \mu}{\sigma} = \frac{28 - 25.9}{2.1} = 1$$

$$\begin{aligned} \Pr(x > 28) &= \Pr(z > 1) \\ &= 1 - 0.84 \text{ [from tables]} \\ &= 0.16 \end{aligned}$$



2. The top 0.5% of students applying for Stato university are given full scholarships. If the mean score on the entrance exam is 372 and the standard deviation is 40 what mark is needed to obtain a scholarship?

First draw a diagram:



In this question we know that $\Pr(x > x_s) = 0.005$ but must work backwards to find the cut off score that defines that area on the graph.

First we find the z-score: $\Pr(z > z_s) = 0.005 \Rightarrow z = 2.58$ [using the tables in reverse]

$$\begin{aligned} \text{Then substituting into the formula } z &= \frac{x - \mu}{\sigma}: 2.58 = \frac{x_s - 372}{40} \\ 2.58 \times 40 &= x_s - 372 \\ 103.2 &= x_s - 372 \\ 103.2 + 372 &= x_s \\ x_s &= 475.2 \end{aligned}$$

Applicants who score more than 475.2 will obtain a scholarship

Exercise

1. If a population has a mean I.Q. of 100 and a standard deviation of 15,
 - (a) find the probability that an individual chosen at random will have an I.Q. between 110 and 130.
 - (b) find the probability that an individual chosen at random will have an I.Q. greater than 87
2. A coffee machine is regulated to deliver 200mL. per cup. In fact, the amount of coffee varies, following a normal distribution with a mean of 200mL. and a standard deviation of 10mL.
 - (a) What is the probability that a cup contain less than 195mL.?
 - (b) What is the probability that a cup will contain more than 220mL.?
 - (c) What is the probability of a cup containing between 195 and 215 mL.?
3. (a) The heights of a group of men follow a normal distribution with a mean of 180 cm. and a standard deviation of 6 cm. What is the probability that a man chosen from this group is less than 185 cm tall?
 - (b) If the tallest 10% of this group are automatically eligible for a basketball team what is the qualifying height.

Answers

1. (a) 0.2297 (b) 0.8069 2. (a) 0.3085 (b) 0.0228 (c) 0.6247 3. (a) 0.7977 (b) 187.69cm